

5.3 : Sum & Difference Identities

think about it!

does $\cos(60^\circ) = \cos(30^\circ) + \cos(30^\circ)$?

$$\begin{aligned} \frac{1}{2} & \stackrel{?}{=} \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \stackrel{?}{=} \frac{2\sqrt{3}}{2} = \sqrt{3} \quad \text{NO!!} \end{aligned}$$

Sum & Difference Identities

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

sign changes!
ex. $\cos(x+y) = \cos x \cos y - \sin x \sin y$

Part I: Evaluating Trig Functions without a calculator

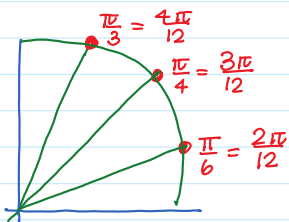
① Find $\cos 15^\circ$ without a calculator

$$\downarrow$$

$$45^\circ - 30^\circ = 15^\circ$$

$$\begin{aligned} \cos(45^\circ - 30^\circ) &= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{2}} \end{aligned}$$

② Find $\sin \frac{7\pi}{12}$ without a calculator



$$\frac{4\pi}{12} + \frac{3\pi}{12} = \frac{7\pi}{12}$$

$$\begin{aligned} \sin\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) &= \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin \frac{\pi}{4} \\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{2}} \end{aligned}$$

③ Find $\sin 165^\circ$ without a calculator

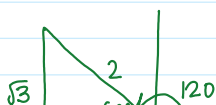
$$\downarrow$$

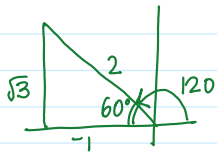
$$120^\circ + 45^\circ = 165^\circ$$

$$\sin(120^\circ + 45^\circ) = \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{2}}$$





$$\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{2}$$

Part II: Write as an expression of sin & cosine

① $\sin 22^\circ \cos 13^\circ + \cos 22^\circ \sin 13^\circ$

$$\sin(22^\circ + 13^\circ) = \sin 35^\circ$$

② $\cos \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \sin \frac{\pi}{4}$

$$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{12}\right)$$

③ $\sin x \sin 3x - \cos x \cos 3x$

$$-\cos x \cos 3x + \sin x \sin 3x = -(\cos x \cos 3x - \sin x \sin 3x)$$

$$-(\cos(x + 3x)) = -\cos 4x$$

