5.1-5.2 Review KEY

Monday, February 9, 2015 9:27 AM

5.1 - 5.2 Review WS

Wednesday, January 22, 2014
1:04 PM

Precalculus
Sections 5.1 - 5.2 (Review)

Name:

PART I: Simplify the following trig expressions.

1) $\sin x \sec x$

$\sin x \cdot \frac{1}{\cos x} = \frac{\sin x}{\cos x} = \tan x$

2) $\sin^2 x\left(\cot^2 x + 1\right)$

$\sin^2 x (\csc^2 x) = \sin^2 x \cdot \frac{1}{\sin^2 x} = 1$

3) $\frac{\csc x}{\cot x}$ $\frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x} = \sec x$

4) $1 - \frac{\sin^2 x}{1 - \cos x}$ $\frac{1-\cos x}{1-\cos x} - \frac{\sin^2 x}{1-\cos x} = \frac{1 - \cos x - \sin^2 x}{1-\cos x}$

$= \frac{1 - \cos x - (1 - \cos^2 x)}{1-\cos x}$

$= \frac{-\cos x + \cos^2 x}{1-\cos x}$

$= \frac{-\cos x (1 - \cos x)}{1-\cos x} = -\cos x$

5) $\sec^2 x + \csc^2 x$

$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{\sin^2 x}{\cos^2 x \sin^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x}$

$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x}$

$$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x}$$

$$= \sec^2 x \cdot \csc^2 x$$

PART II: Prove the following trig identities

6) $\csc x - \cos x \cot x = \sin x$

$$\frac{1}{\sin x} - \cos x \cdot \frac{\cos x}{\sin x}$$

$$\frac{1 - \cos^2 x}{\sin x}$$

$$\frac{\sin^2 x}{\sin x} = \sin x \checkmark$$

7) $\dfrac{\sec x + \tan x}{\csc x + 1} = \tan x$

$$\frac{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}{\frac{1}{\sin x} + 1} = \frac{\frac{1 + \sin x}{\cos x}}{\frac{1 + \sin x}{\sin x}} = \frac{1 + \sin x}{\cos x} \cdot \frac{\sin x}{1 + \sin x} = \frac{\sin x}{\cos x} = \tan x \checkmark$$

8) $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$

$$\frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cdot \cos^2 x}{\cos^2 x} = \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} = \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$$

$$= \frac{\sin^2 x \cdot \sin^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x} \cdot \sin^2 x = \tan^2 x \sin^2 x$$

9) $\dfrac{\sin x}{1 + \cos x} = \dfrac{1 - \cos x}{\sin x}$

$$\frac{\sin x}{1 + \cos x} \cdot \frac{(1 - \cos x)}{(1 - \cos x)} = \frac{\sin x (1 - \cos x)}{1 - \cos^2 x} = \frac{\sin x (1 - \cos x)}{\sin^2 x} = \frac{1 - \cos x}{\sin x}$$

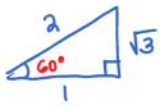
$\frac{\sin x}{1+\cos x} \cdot \frac{(1-\cos x)}{(1-\cos x)} = \frac{\sin x(1-\cos x)}{1-\cos^2 x} = \frac{\sin x(1-\cos x)}{\sin^2 x} = \boxed{\frac{1-\cos x}{\sin x}}$

PART III: Solve the following trig equations over [0,2π)

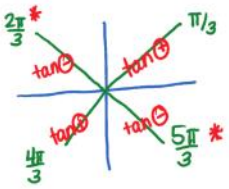
10) $\tan x + \sqrt{3} = 0$

$\tan x = -\sqrt{3}$

$\frac{\sin x}{\cos x} = \frac{-\sqrt{3}}{1}$



$60° = \frac{\pi}{3}$



So $\boxed{x = \frac{2\pi}{3}, \frac{5\pi}{3}}$

11) $2\sin x \cos x = \sqrt{2}\cos x$

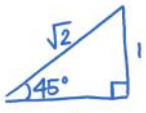
$2\sin x\cos x - \sqrt{2}\cos x = 0$

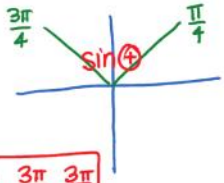
$\cos x(2\sin x - \sqrt{2}) = 0$

$\cos x = 0 \quad 2\sin x - \sqrt{2} = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \sin x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$



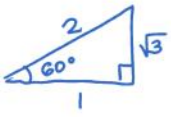


So $\boxed{x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}}$

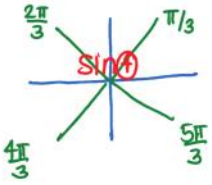
12) $4\sin^2 x = 3$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$



$60° = \frac{\pi}{3}$



$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

13) $\tan x \sec x = \tan x$

$\tan x\sec x - \tan x = 0$

$\tan x(\sec x - 1) = 0$

$\tan x = 0 \quad \sec x - 1 = 0$

$\frac{\sin x}{\cos x} = 0 \quad \sec x = 1$

$x = 0, \pi \quad \frac{1}{\cos x} = 1$

$x = 0$

$\boxed{x = 0, \pi}$

14) $2\sec^2 x - 3\sec x - 2 = 0$

$(2\sec x + 1)(\sec x - 2) = 0$

$2\sec x + 1 = 0 \quad \sec x = 2$

$\sec x = -\frac{1}{2} \quad \frac{1}{\cos x} = \frac{2}{1}$

$2\sec x + 1 = 0$

$\sec x = -\frac{1}{2}$

$\frac{1}{\cos x} = \frac{1}{-2}$

$\cos x = -2$

$x = \emptyset$

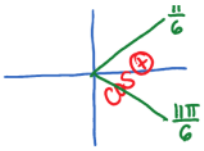
$\sec x = 2$

$\frac{1}{\cos x} = \frac{2}{1}$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

$30° = \frac{\pi}{6}$



$x = \frac{\pi}{6}, \frac{11\pi}{6}$

15) $\tan^2 x - \sec x - 1 = 0$

$(\sec^2 x - 1) - \sec x - 1 = 0$

$\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x = 2$

$\frac{1}{\cos x} = \frac{2}{1}$

$\cos x = \frac{1}{2}$

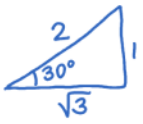
$x = \frac{\pi}{6}, \frac{11\pi}{6}$

$\sec x = -1$

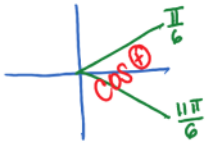
$\frac{1}{\cos x} = \frac{-1}{1}$

$\cos x = -1$

$x = \pi$



$30° = \frac{\pi}{6}$



$x = \frac{\pi}{6}, \pi, \frac{11\pi}{6}$