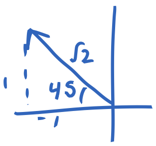
Precalculus
Q3 Cumulative Review
*Problems marked with ** are calculator OK.*

Name: Key
Period:

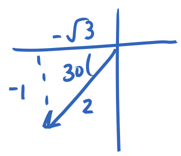
PART I: 4.1-4.3, 5.5, 5.6 *(Right Angle Trig, Unit Circle, Law of Sines & Cosines)*

(#1-6) Without using a calculator, find the exact values of each:

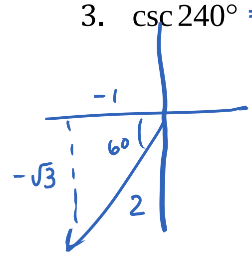
1. $\cos 135° = \boxed{-\frac{\sqrt{2}}{2}}$



2. $\cot \frac{7\pi}{6} = \boxed{\sqrt{3}}$



3. $\csc 240° = \boxed{\frac{-2\sqrt{3}}{3}}$

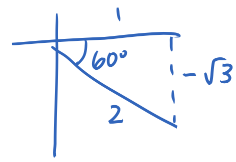


4. $\sin \pi = \boxed{0}$

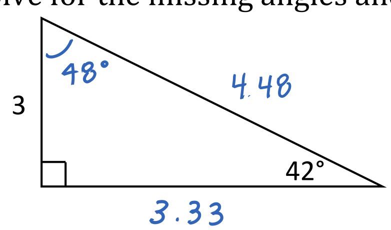
5. $\sec 90° = \boxed{\text{undefined}}$

$\frac{1}{x}$

6. $\tan \frac{5\pi}{3} = \boxed{-\sqrt{3}}$



7. Solve for the missing angles and sides of the triangle. **



$\sin 42 = \frac{3}{h}$

$h = 4.48$

$\cos 42 = \frac{x}{4.48}$

$x = 3.33$

8. Convert 37 degrees to radians.**

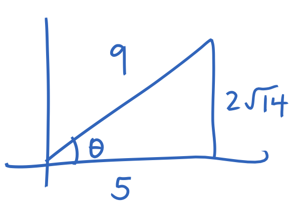
$37° \times \frac{\pi}{180°} = \boxed{\frac{37\pi}{180}}$

9. Convert 2 radians to degrees.**

$2 \times \frac{180°}{\pi} = \boxed{\frac{360°}{\pi} \text{ or } 114.59°}$

10. Assume the angle θ is an acute angle. Find the other five trig. functions if:

a. $\cos\theta = \frac{5}{9}$



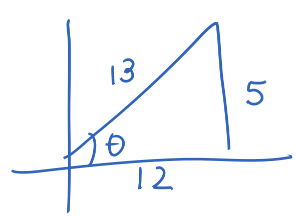
$5^2 + y^2 = 9^2$
$y^2 = 56$
$y = 2\sqrt{14}$

$\sin\theta = \frac{2\sqrt{14}}{9}$ $\csc\theta = \frac{9}{2\sqrt{14}}$

$\sec\theta = \frac{9}{5}$

$\tan\theta = \frac{2\sqrt{14}}{5}$ $\cot\theta = \frac{5}{2\sqrt{14}}$

b. $\csc\theta = \frac{13}{5}$

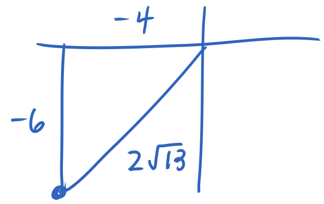


$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$ $\sec\theta = \frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\cot\theta = \frac{12}{5}$

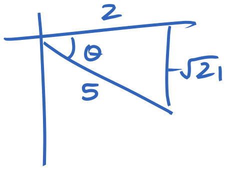
11. Evaluate the six trig. functions if point P (-4, -6) is on the terminal side of an angle θ.



$4^2+6^2=h^2$
$52=h^2$
$2\sqrt{13}=h$

$$\boxed{\begin{array}{ll} \sin\theta = \frac{-6}{2\sqrt{13}} = \frac{-3}{\sqrt{13}} & \csc\theta = -\frac{\sqrt{13}}{3} \\ \cos\theta = \frac{-4}{2\sqrt{13}} = \frac{-2}{\sqrt{13}} & \sec\theta = -\frac{\sqrt{13}}{2} \\ \tan\theta = \frac{-6}{-4} = \frac{3}{2} & \cot\theta = \frac{2}{3} \end{array}}$$

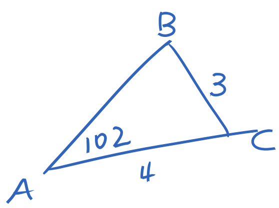
12. Find $\sin\theta$ and $\tan\theta$ if $\underbrace{\cos\theta = \frac{2}{5}}_{\text{quad 1 or 4}}$ and $\underbrace{\cot\theta < 0}_{\text{quad 2 or 4}}$.



$$\boxed{\sin\theta = -\frac{\sqrt{21}}{5} \qquad \tan\theta = -\frac{\sqrt{21}}{2}}$$

13. Determine if the triangle has 0, 1, or 2 possible triangles. **

a. a = 3, b = 4, A = 102°



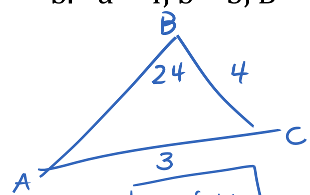
$\frac{\sin 102}{3} = \frac{\sin B}{4}$

$\sin B = \frac{4\sin 102}{3}$

$\sin B = 1.30$

$\boxed{\text{No } \Delta\text{'s}}$

b. a = 4, b = 3, B = 24°



$\boxed{2\ \Delta\text{'s}}$

$\frac{\sin 24}{3} = \frac{\sin A}{4}$

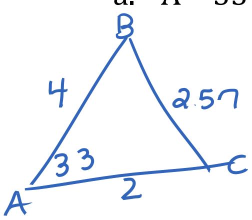
$\sin A = \frac{4\sin 24}{3}$

$A = 32.84$

supp. of $A = 147.16$

14. Solve the triangle and find the area of the triangle. **

a. A = 33°, b = 2, c = 4



$a^2 = 4^2+2^2-2(4)(2)\cos 33$

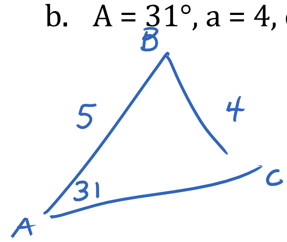
$\boxed{a = 2.57}$

$\frac{\sin 33}{2.57} = \frac{\sin B}{2}$

$\boxed{B = 25.13°}$ $\boxed{C = 121.87°}$

Area $= \frac{1}{2}(4)(2)\sin 33$
$= 2.18\text{ units}^2$

b. A = 31°, a = 4, c = 5



$\frac{\sin 31}{4} = \frac{\sin C}{5}$

$\boxed{\begin{array}{l} C = 40.08° \\ B = 108.92° \end{array}}$

$\frac{\sin 108.92}{b} = \frac{\sin 31}{4}$

$\boxed{b = 7.35}$

$\boxed{\begin{array}{l} \text{Area} = \frac{1}{2}(5)(4)\sin 108 \\ \text{Area} = 9.46\text{u}^2 \end{array}}$

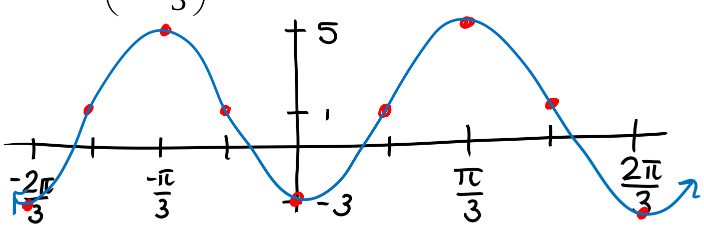
Δ #2

$$\boxed{\begin{array}{l} \angle C = 139.92° \\ \angle B = 9.08° \\ b = 1.23 \end{array}}$$

PART II: 4.4, 4.5, 4.7 *(Trig Graphs, Inverse Trig, Solving Trig Equations)*

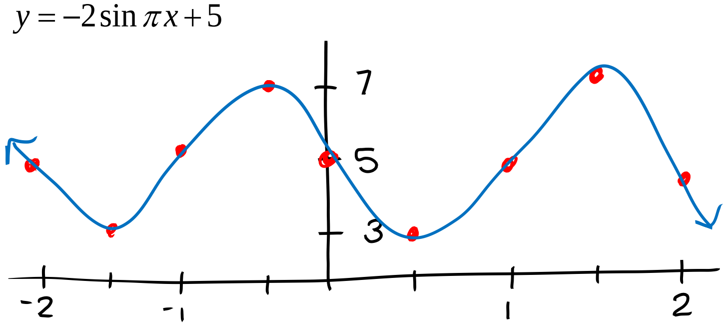
(#15-18) Sketch two periods of the graph of the trig function. Make sure to include your scale and critical values on each axis.

15. $y = 4\cos 3\left(x - \frac{\pi}{3}\right) + 1$



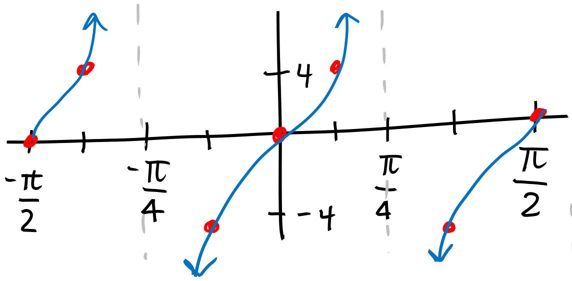
$P = \frac{2\pi}{3}$

16. $y = -2\sin \pi x + 5$



$P = \frac{2\pi}{\pi} = 2$

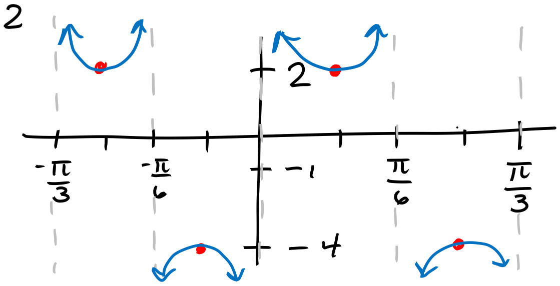
17. $y = 4\tan 2x$



$P = \frac{\pi}{2}$

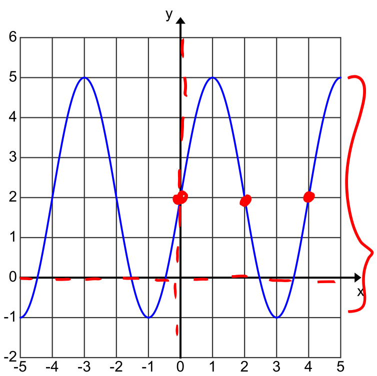
18. $y = 3\csc 6x - 1$

$\longrightarrow$ $P = \frac{2\pi}{6} = \frac{\pi}{3}$



(#19-20) Write the equation of the each graph shown below. (4 points each)

19.



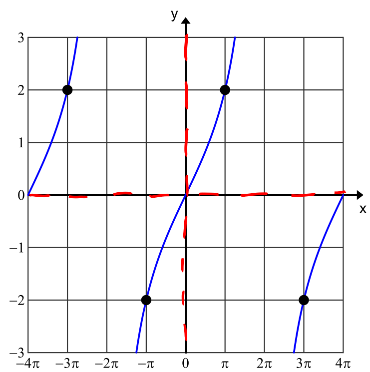
$a = 3$

$p = 4$

$\frac{2\pi}{b} = 4 \Rightarrow b = \frac{\pi}{2}$

$y = 3\sin\frac{\pi}{2}x + 2$

20.



$P = 4\pi$

$4\pi = \frac{\pi}{b}$

$b = 1/4$

$y = 2\tan\frac{1}{4}x$

(#21-25) Solve the trig equation over the interval $[0, 2\pi]$.

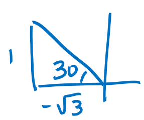
21. $\cos x = -\frac{\sqrt{2}}{2}$

$\cancel{x} = 45° \Rightarrow$ quad 2 & 3

$x = \frac{3\pi}{4}, \frac{5\pi}{4}$

22. $\cot x = -\sqrt{3}$

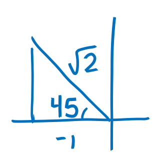
quad 2 & 4



$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

23. $\sec x = -\sqrt{2}; [-\pi, \pi]$

quad 2 & 3



$x = \frac{3\pi}{4}, \frac{5\pi}{4}$

24. $\sin x = 0.73$ **

quad 1 & 2

$x = .82, 2.32$

25. $\sec x = -1.92$ **

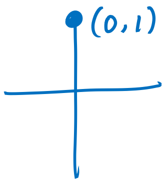
quad 2 & 3

$\cos^{-1}(1/-1.92)$

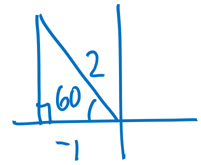
$x = 2.12, 4.16$

(#26-30) evaluate the inverse trig function. *Reminder: inverse trig functions have restricted domains!*

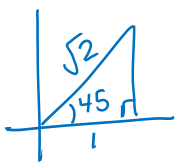
26. $\sin^{-1}(1) = \frac{\pi}{2}$



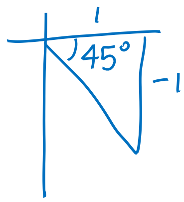
27. $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$



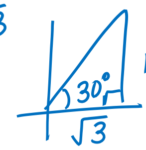
28. $\sec^{-1}\left(\sqrt{2}\right) = 45°$



29. $\tan^{-1}(-1) = -\frac{\pi}{4}$



30. $\cos\left(\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right)$ $\frac{1}{\sqrt{3}}$



$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

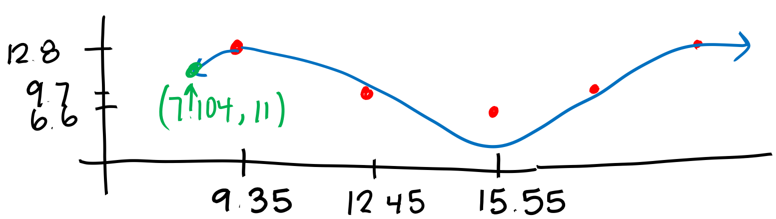
31. Write the equation of a sine graph that has an amplitude of 4, a period of length 3π, a phase shift of $\frac{\pi}{4}$ to the left, and a vertical shift down 2. ↓2

$a = 4$ $\qquad 3\pi = \frac{2\pi}{b}$ $\qquad + \frac{\pi}{4}$

$b = \frac{2}{3}$

$y = 4\sin\frac{2}{3}\left(x + \frac{\pi}{4}\right) - 2$

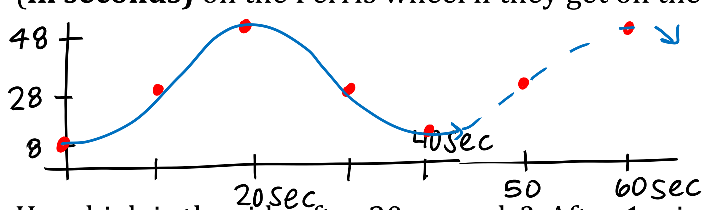
32. At Hilton Head Island on June 10, high tide measured 12.8 feet on a pier at 9:21 am. The next low tide measured 6.6 feet at 3:33 pm. Write a sinusoidal equation modeling the behavior of the tide. What is the first time on June 10 that the tide measures 11 feet? ** $P = 12.4 = \frac{2\pi}{b} = \frac{\pi}{6.2}$



$y = 3.1\cos\frac{\pi}{6.2}(x - 9.35) + 9.7$

approximately 7:06am

33. The Ferris wheel at a local amusement park has a diameter of 40 feet and reaches a maximum height of 48 feet above the ground. One ride is three revolutions, which takes 2 minutes to complete.**
   a. Draw a sketch of the graph and create an equation to model the height of a rider in terms of time **(in seconds)** on the Ferris wheel if they get on the ride at the bottom.



$\frac{2}{3} = \frac{2\pi}{b}$ $\qquad \frac{2\text{ min}}{3\text{ rev}} = \frac{2}{3}\text{ min/rev} \approx 40\text{sec}$

$b = 3\pi$

$y = -20\cos 3\pi x + 28$

   b. How high is the ride after 20 seconds? After 1 minute?

48 feet

   c. At what time(s) during the full ride does the rider reach a height of 25 feet?

.15, .52, .82, 1.18, 1.48, 1.85

9sec, 31.2sec, 49.2sec, 70.8sec, 88.8sec, 111sec

PART III: Chapter 5 *(Trig Identities)*

(#34-37) Simplify using trig identities:

34. $\cos^3 x + \cos x \sin^2 x$

$\cos x(\underbrace{\cos^2 x + \sin^2 x}_{=1})$

$\cos x$

35. $\dfrac{\cos^2 u + \cot^2 u + \sin^2 u}{\csc u}$

$=1$

$\dfrac{1+\cot^2 u}{\frac{1}{\sin u}} = \csc^2 u \cdot \sin u$

$\dfrac{1}{\sin^2 u} \cdot \cancel{\sin} u$

$\dfrac{1}{\sin u} \Rightarrow \boxed{\csc u}$

36. $\dfrac{1}{\sin^2 x} + \dfrac{\sec^2 x}{\tan^2 x}$

$\dfrac{1}{\sin^2 x} + \dfrac{1}{\cancel{\cos^2 x}} \cdot \dfrac{\cancel{\cos^2 x}}{\sin^2 x}$

$\dfrac{2}{\sin^2 x} = 2\csc^2 x$

37. $\dfrac{1+\cot\theta}{1+\tan\theta} = \dfrac{\frac{\sin x}{\sin x} + \frac{\cos x}{\sin x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}$

$= \dfrac{\sin x + \cos x}{\sin x} \cdot \dfrac{\cos x}{\cos x + \sin x}$

$\cot x$

(#38-41) Find all solutions in the interval $[0,2\pi)$.

38. $\sqrt{2}\cot x \sin x - \cot x = 0$

$\cot x(\sqrt{2}\sin x - 1) = 0$

$\cot x = 0$ $\qquad$ $\sin x = 1/\sqrt{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $x = \frac{\pi}{4}, \frac{3\pi}{4}$

39. $3\cos t = 2\sin^2 t$

$3\cos t - 2(1-\cos^2 t) = 0$

$2\cos^2 t + 3\cos t - 2 = 0$

$(2\cos t - 1)(\cos t + 2) = 0$

$\cos t = 1/2$ $\qquad$ $\cos t = \cancel{-2}$

$t = \frac{\pi}{3}, \frac{5\pi}{3}$

40. $\sin 2x - 2\sin x = 0$

$2\sin x\cos x - 2\sin x = 0$

$2\sin x(\cos x - 1) = 0$

$2\sin x = 0$ $\qquad$ $\cos x = 1$

$x = 0, \pi$ $\qquad$ $x = 0$

41. $\cos 2x = \sin x$

$\cos 2x - \sin x = 0$

$(1 - 2\sin^2 x) - \sin x = 0$

$-2\sin^2 x - \sin x + 1 = 0$

$-(2\sin^2 x + \sin x - 1) = 0$

$-(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = 1/2$ $\qquad$ $\sin x = -1$

(#42-45) Prove the following:

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad x = \frac{3\pi}{2}$

42. $\cos x + \sec x = \dfrac{2 - \sin^2 x}{\cos x}$

$\frac{\cos x}{\cos x} \cos x + \frac{1}{\cos x}$

$\frac{\cos^2 x + 1}{\cos x} = \frac{(1 - \sin^2 x) + 1}{\cos x}$

$= \frac{2 - \sin^2 x}{\cos x}$ ✓

43. $1 + \tan^2 x = \dfrac{1}{1 - \sin^2 x}$

$= \sec^2 x$

$= \frac{1}{\cos^2 x}$

$= \frac{1}{1 - \sin^2 x}$ ✓

44. $\dfrac{1}{1 - \cos t} = \dfrac{1 + \cos t}{\sin^2 t}$

$= \frac{(1 + \cos t)\, 1}{(1 + \cos t)(1 - \cos t)}$

$= \frac{1 + \cos t}{1 - \cos^2 t} = \frac{1 + \cos t}{\sin^2 t}$ ✓

45. $\cos\left(x - \dfrac{3\pi}{2}\right) = -\sin x$

$= \cos x \cos \frac{3\pi}{2} + \sin x \sin \frac{3\pi}{2}$

$= \cos x\,(0) + \sin x\,(-1)$

$= -\sin x$ ✓

(#46-49) Evaluate the following without a calculator, using either the Sum/Difference or Half-Angle identities.

46. $\sin 105°$

$\sin(60 + 45)$

$\sin 60 \cos 45 + \cos 60 \sin 45$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6} + \sqrt{2}}{4}$

47. $\cos(-75°)$

$\cos(60 - 135)$

$\cos(60)\cos(135) + \sin(60)\sin(135)$

$\left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\frac{-\sqrt{2} + \sqrt{6}}{4}}$

48. $\tan\left(\dfrac{\pi}{12}\right)$

$\tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$\frac{\tan \frac{\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{3} \tan \frac{\pi}{4}} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$

49. $\sin\left(\dfrac{5\pi}{8}\right)$

$\sin\left(\frac{\frac{5\pi}{4}}{2}\right) = +\sqrt{\frac{1 - \cos \frac{5\pi}{4}}{2}}$

$= \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$