Precalculus: 2nd Semester Review NAME Key

** *Marks calculator OK*

Chapter 4.1-4.3, 5.5, 5.6 (Right triangle and unit circle trig., Law of Sines and Cosines)

1. Find the exact values of each:

a) $\sin 300° = \boxed{-\frac{\sqrt{3}}{2}}$

b) $\cot \frac{7\pi}{6} = \boxed{\sqrt{3}}$

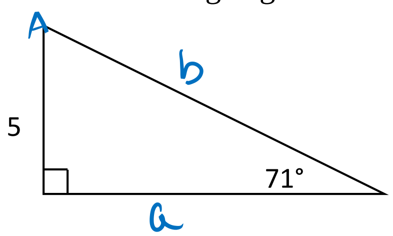
c) $\sec 225° = \boxed{-\sqrt{2}}$

d) $\tan \pi = \boxed{0}$ (-1, 0)

e) $\csc \frac{\pi}{2} = \boxed{1}$ (0, 1)

f) $\tan \frac{5\pi}{3} = \boxed{-\sqrt{3}}$

**2. Solve for missing angles and sides of the triangle:



$A = 19°$

$\sin 71° = \frac{5}{b}$

$b = \frac{5}{\sin 71°} \approx \boxed{5.288}$

$\tan 71° = \frac{5}{a}$

$a = \frac{5}{\tan 71°} \approx \boxed{1.722}$

3. Convert 108 degrees to radians.

$108°\left(\frac{\pi}{180°}\right) = \boxed{\frac{3\pi}{5}}$

4. Convert 10 radians to degrees.

$10\left(\frac{180°}{\pi}\right) = \boxed{\frac{1800°}{\pi}}$

5. Evaluate the six trig. functions if point P (-8, 3) is on the terminal side of an angle θ.

P, 3, -8, $\sqrt{73}$, θ

$\sqrt{8^2+3^2} = \sqrt{73}$

$\sin\theta = \frac{3}{\sqrt{73}}$ $\quad \tan\theta = -\frac{3}{8}$ $\quad \csc\theta = \frac{\sqrt{73}}{3}$

$\cos\theta = \frac{-8}{\sqrt{73}}$ $\quad \cot\theta = -\frac{8}{3}$ $\quad \sec\theta = -\frac{\sqrt{73}}{8}$

6. Find $\sin\theta$ and $\cot\theta$ if $\cos\theta = \frac{3}{7}$ and $\tan\theta < 0$.

Cos + $\quad$ tan −

Q1, 4 $\quad$ Q2, 4

3, 7, $-2\sqrt{10}$

$\sin\theta = -\frac{2\sqrt{10}}{7}$

$\cot\theta = -\frac{3}{2\sqrt{10}}$

**7. Determine if the triangle has 0, 1, or 2 possible triangles:

a) $m\angle C = 40°,\ a = 8,\ c = 4$

8, 4, h, 40°

$h = 8\sin 40 = 5.14 > 4$

$\boxed{\text{0 triangles}}$

b) $m\angle H = 50°,\ a = 12,\ h = 10$

12, 10, h, 50°

$h = 12\sin 50° = 9.19 < 10$

$\boxed{\text{2 triangles}}$

**8. Solve the triangle and find the area of the triangle:

a) A = 88°, b = 6, c = 7 SAS- Cosines

C, A, B, 6, 7, a, 88°

$a^2 = 6^2 + 7^2 - 2(6)(7)\cos 88°$

$\boxed{a = 9.059}$

$\frac{\sin 88}{9.059} = \frac{\sin B}{6}$

$\boxed{\angle B = 41.45°}$ $\quad \boxed{\angle C = 50.55°}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2} = 11.029$

$\boxed{A = 20.987}$

b) t = 9, a = 4, c = 6 SSS -Cosines

T, A, C, 6, 4, 9

$9^2 = 4^2 + 6^2 - 2(4)(6)\cos T$

$81 = 52 - 48\cos T$

$T = \cos^{-1}\left(\frac{-29}{48}\right)$

$\boxed{T = 127.17°}$

$A = \frac{1}{2}(4)(9)\sin C$

$\boxed{A = 9.56}$

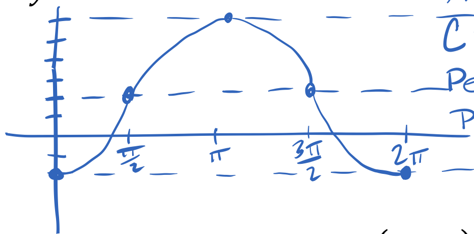
$\frac{\sin 127.17°}{9} = \frac{\sin A}{4}$

$\boxed{\angle A = 20.74°}$ $\quad \boxed{\angle C = 32.09°}$

Sections 4.4, 4.5, 4.7 (Trig Graphs, Inverse Trig)

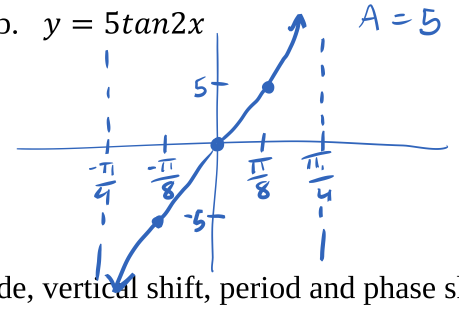
9. Sketch the following graphs and describe their characteristics:

a. $y = 2 - 4\cos x$



A = Amplitude = 4
C = Sinusoidal Axis = 2
Period = 2π
Phase Shift = 0

b. $y = 5\tan 2x$



A = 5 B = 2
Per = $\frac{\pi}{2}$
Crit. Values $\frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$

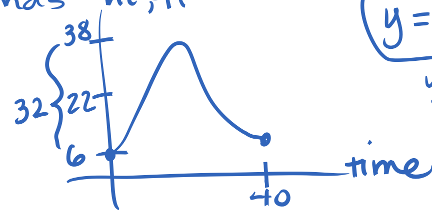
10. For the equation $y = -5\sin\frac{\pi}{3}\left(x + \frac{\pi}{4}\right) - 9$, find the amplitude, vertical shift, period and phase shift.

Amplitude = 5
Vertical shift = −9
$B = \frac{\pi}{3}$ Period = $\frac{2\pi}{\pi/3} = 2\pi \cdot \frac{3}{\pi} = 6$

Phase Shift = $-\frac{\pi}{4}$

11. **A Ferris wheel 32 feet in diameter makes one revolution every 40 seconds. If the bottom of the wheel is 6 feet above the ground, write the equation to model the path of the rider on the wheel. Then, use the equation to find the first three times a rider is 25 feet above the ground (assume he gets on at the bottom).**

Period = 40 seconds
$B = \frac{2\pi}{40} = \frac{\pi}{20}$
radius = A = 16
C = 22
D = 0



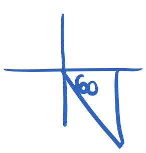
$\boxed{y = -16\cos\left(\frac{\pi}{20}x\right) + 22}$

$y_2 = 25$

$\boxed{t = 11.2, 28.8, 51.2 \text{ seconds}}$

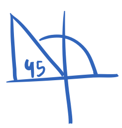
12. Find the following (give answers in radians):

$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$



$\boxed{-\frac{\pi}{3}}$

$\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)$



$\boxed{\frac{3\pi}{4}}$

$\tan^{-1}(-\sqrt{3})$

$\boxed{-\frac{\pi}{3}}$

$\cot^{-1}(-1)$

$\boxed{\frac{3\pi}{4}}$

Chapter 5 (Trig Identities, Solving Trig Equations)

13. Simplify the following:

a. $\frac{\cos^2 x + \sin^2 x}{\tan x} = \frac{1}{\tan x} = \boxed{\cot x}$

b. $\frac{1 + \tan^2 x}{\csc^2 x} = \frac{\sec^2 x}{\csc^2 x} = \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$

14. Solve the following over $[0, 2\pi)$:

a. $1 - \sqrt{2}\cos x = 0$

$-\sqrt{2}\cos x = -1$
$\cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{7\pi}{4}}$

b. $2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$
$\sin x = \frac{1}{2}$ $\sin x = -1$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

c. $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$
$\cos x(2\sin x - 1) = 0$
$\cos x = 0$ $2\sin x - 1 = 0$
$\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$

d. $3\tan^2 x - 1 = 0$

$\tan^2 x = \frac{1}{3}$
$\tan x = \frac{\pm 1}{\sqrt{3}} = \frac{\pm\sqrt{3}}{3}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

15. Prove the following:

a. $\csc x - \sin x = \cot x \cos x$

$\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} =$

$\frac{\cos^2 x}{\sin x} =$

✓ $\cot x \cdot \cos x = \cot x \cos x$

b. $\cot x(\tan x \sin x + \cos x) = \csc x$

$\cot x \cdot \tan x \sin x + \cot x \cdot \cos x =$

$1 \cdot \sin x + \frac{\cos x}{\sin x} \cdot \cos x =$

$\sin x + \frac{\cos^2 x}{\sin x} =$

$\frac{\sin^2 x + \cos^2 x}{\sin x} =$

$\frac{1}{\sin x} =$

✓ $\csc x = \csc x$

c. $\frac{(1-\cos x)\sin x}{(1-\cos x)(1+\cos x)} = \csc x - \cot x$

$\frac{(1-\cos x)\sin x}{1-\cos^2 x} =$

$\frac{(1-\cos x)\sin x}{\sin^2 x} =$

$\frac{1-\cos x}{\sin x} =$

$\frac{1}{\sin x} - \frac{\cos x}{\sin x} =$

✓ $\csc x - \cot x = \csc x - \cot x$

d. $\cos 3x = \cos^3 x - 3\sin^2 x \cdot \cos x$

$\cos(2x+x) =$

$\cos 2x \cos x - \sin 2x \sin x =$

$(\cos^2 x - \sin^2 x)\cos x - 2\sin x \cos x \sin x =$

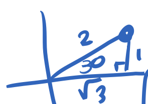
$\cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x =$

$\cos^3 x - 3\sin^2 x \cos x$ ✓

Chapter 6 (Vectors, Parametric Equations, Polars, Complex Plane)

16. Convert the following coordinates from polar to rectangular or from rectangular to polar:

a. $(3,30°)$



$(3\cos 30°, 3\sin 30°)$

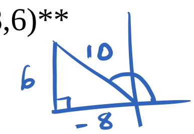
$\boxed{\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)}$

b. $(-8,265°)$**

$(-8\cos 265°, -8\sin 265°)$

$= \boxed{(.70, 7.97)}$

c. (-8,6)**

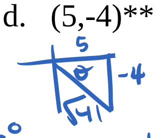


$x = \tan^{-1}\left(\frac{6}{8}\right) = 36.87°$

$\theta = 180 - x = 143.13°$

$\boxed{(10, 143.13°)}$

d. (5,-4)**



$\theta = \tan^{-1}\left(\frac{-4}{5}\right)$

$\theta = -38.66$

$\boxed{(\sqrt{41}, -38.66°)}$

17. **At the driving range, a golf ball is struck from an elevated tee box 8 feet above the ground. The ball was struck with an initial velocity of 180 ft./sec. at an angle of 38° to the horizontal.

a. Write the parametric equations for the flight of the ball.

$X = 180\cos(38°)T$

$Y = -16T^2 + 180\sin(38°)T + 8$

b. Find when and where the ball will be at its highest point.

199.89 ft high

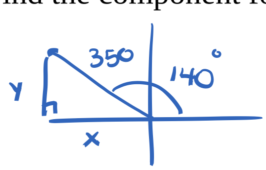
at $t = 3.4$ sec

c. Determine when and where the ball will strike the ground.

$t = 7$ sec

$x = 992.5$ ft

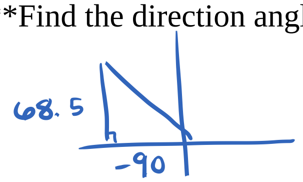
18. **Find the component form of a vector with a magnitude of 350 and a direction angle of 140°.



$\langle 350\cos 140°, 350\sin 140° \rangle$

$\boxed{\langle -268.12, 224.98 \rangle}$

19. **Find the direction angle and magnitude of a vector with a component form $\langle -90, 68.5 \rangle$.



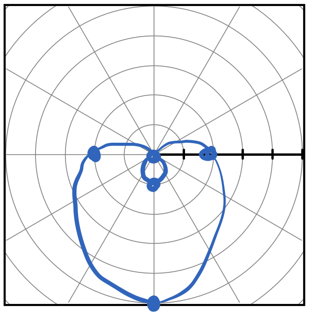
$|v| = \sqrt{90^2 + 68.5^2} = \boxed{113.10}$

$x = \tan^{-1}\left(\frac{68.5}{90}\right) = 37.28°$

$\theta = 180 - x = \boxed{142.72°}$

20. Graph the following polar equations.

a. $r = 2 - 3\sin\theta$ Limaçon

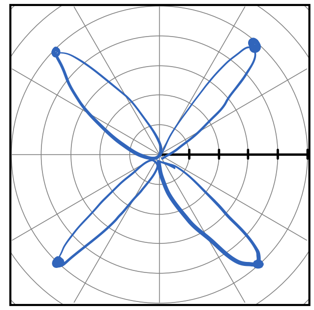


b. $r = 5\sin(2\theta)$ Rose

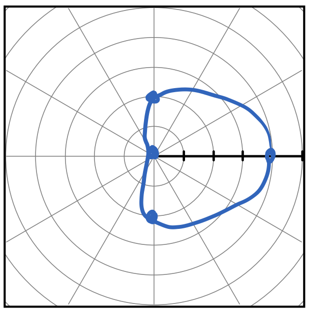
$\frac{90}{n} = 45°$

4 petals

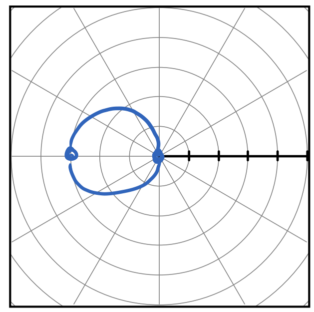
$\frac{360°}{4} = 90°$



a. $r = 2 + 2\cos\theta$ Cardioid



b. $r = -3\cos\theta$ Circle

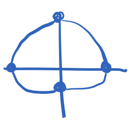


21. Write the equation to match the polar described below:

a. Write the equation of a limaçon that has x-intercepts at 4 and -4, and y-intercepts at 5 and -3.

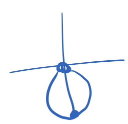
$a = 4$ $\quad a + b = 5$

$b = 1$



$\boxed{r = 4 + 1\sin\theta}$

b. Write the equation of a circle that lies on the negative y-axis, with y-intercepts of 0 and -10.



$\boxed{r = -10\sin\theta}$

22. Eliminate the Parameter: $x = 2t^2 + 3$; $y = t - 1$

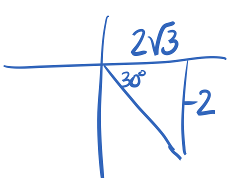
$x - 3 = 2t^2$

$\frac{x-3}{2} = t^2$

$t = \pm\sqrt{\frac{x-3}{2}}$

$\boxed{y = \pm\sqrt{\frac{x-3}{2}} - 1}$

23. Use De Moivre's Theorem to simplify $\left(2\sqrt{3} - 2i\right)^4$. Express your answer in standard form.



$r = \sqrt{2^2 + (2\sqrt{3})^2}$

$r = \sqrt{4 + 12}$

$r = 4$

$\theta = \frac{11\pi}{6}$

$= \left(4 \text{cis} \frac{11\pi}{6}\right)^4 = \left[4\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)\right]^4$

$= 4^4 \text{cis} \frac{44\pi}{6} = 4^4 \text{cis} \frac{22\pi}{3}$

$= \boxed{256 \text{cis} \frac{4\pi}{3}}$ ← (same terminal side)

$= 256\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$= 256\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = \boxed{-128 - 128i\sqrt{3}}$

Conics

24. Convert the following parametric equation of a conic section into general form: $x = 2 + 3\cos t$, $y = 4 + 6\sin t$

center $(2,4)$ ellipse $a=6$ $b=3$

$$\boxed{\frac{(x-2)^2}{9} + \frac{(y-4)^2}{36} = 1}$$

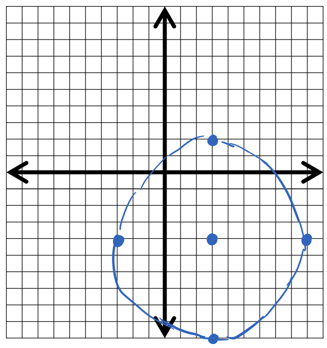
25. Determine the shape of the conic section and identify all important information before graphing. (If necessary, convert the following equations into general form by completing the square.)

a. $x^2 - 6x + y^2 + 8y - 11 = 0$ Circle

$x^2 - 6x + \underline{9} + y^2 + 8y + \underline{16} = 11 + \underline{9} + \underline{16}$

$\boxed{(x-3)^2 + (y+4)^2 = 36}$

center $(3,-4)$
$r = 6$



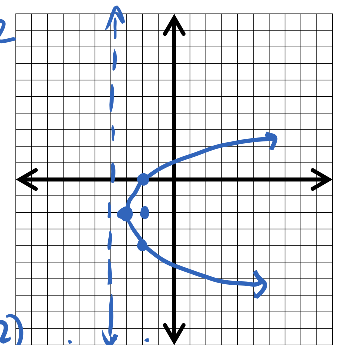
b. $y^2 + 4y - 4x - 8 = 0$ Parabola

$y^2 + 4y + \underline{4} = 4x + 8 + \underline{4}$

$(y+2)^2 = 4x + 12$

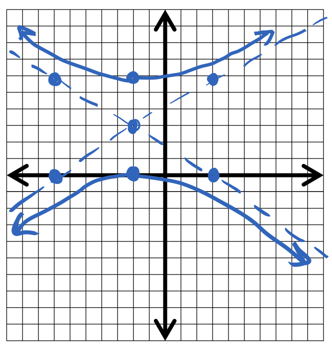
$(y+2)^2 = 4(x+3)$

vertex $(-3,-2)$
$4p = 4$
$p = 1$
opens Right
focus $(-2,-2)$
Directrix $x = -4$ fw=4



c. $\frac{(y-3)^2}{9} - \frac{(x+2)^2}{25} = 1$ Hyperbola

center $(-2,3)$
opens up/down
$a = 3$
$b = 5$
slope of asymptotes $= \pm\frac{3}{5}$
foci $f^2 = 25 + 9 = 34$
$f = \sqrt{34}$
coords $= (-2, 3 \pm \sqrt{34})$



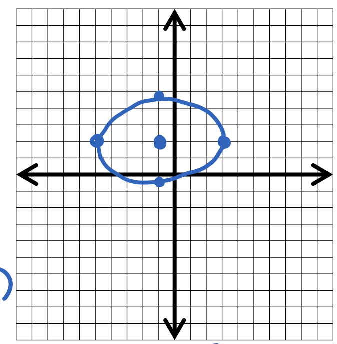
d. $x^2 + 2x + 2y^2 - 8y = 7$ Ellipse

$x^2 + 2x + \underline{1} + 2(y^2 - 4y + \underline{4}) = 7 + \underline{1} + \underline{8}$

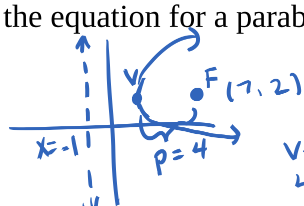
$\frac{(x+1)^2}{16} + \frac{2(y-2)^2}{16} = \frac{16}{16}$

$\frac{(x+1)^2}{16} + \frac{(y-2)^2}{8} = 1$

center $(-1,2)$
$a = 4$ (horizontal)
$b = \sqrt{8}$
foci : $f^2 = 16 - 8 = 8$
$f = \sqrt{8} = 2\sqrt{2}$
coords $(-1 \pm 2\sqrt{2}, 2)$



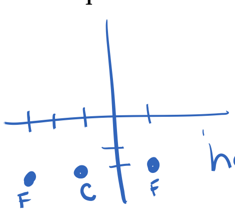
26. Find the equation for a parabola with focus at (7, 2) and directrix x = -1.



vertex $(3,2)$
$4p = 16$ opens right

$(y-2)^2 = 16(x-3)$

27. Find the equation of an ellipse with foci at (-3, -2) and (1, -2) and major axis length = 8.



horizontal

center $(-1,-2)$
$f = 2$
$f^2 = a^2 - b^2$
$2^2 = 4^2 - b^2$
$b^2 = 12$

$2a = 8$
$a = 4$

$$\boxed{\frac{(x+1)^2}{16} + \frac{(y+2)^2}{12} = 1}$$