Day 3 Notes

Precalculus   Name:
Conics – Ellipses – Day 1 Notes   Period:

LET'S TAKE IT UP A NOTCH!

1. Write equation of ellipse with foci at $(0, \pm 5)$ and major axis length = 26.

13 13 5 x 12

$$\frac{x^2}{144} + \frac{y^2}{169} = 1$$
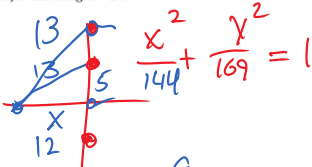
2. Write equation of ellipse with endpoints of the major axis at $(\pm 6, 0)$ and endpoints of the minor axis at $(0, \pm 5)$.

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

6   5
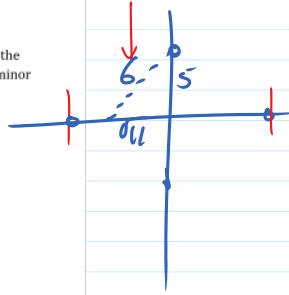
3. Eliminate the parameter. $x = -3 + 5\cos t$
                               $y = 3 + 2\sin t$

Center $(-3, 3)$

$a = 5$

$b = 2$

$$\frac{(x+3)^2}{25} + \frac{(y-3)^2}{4} = 1$$

COMPLETING THE SQUARE FOR AN ELLIPSE

4. $3x^2 + 5y^2 - 12x + 30y + 42 = 0$

$3x^2 - 12x + \_\_ + 5y^2 + 30y + \_\_ = -42 + \_\_ + \_\_$

$3(x^2 - 4x + 4) + 5(y^2 + 6y + 9) = -42 + 12 + 45$

$3(x-2)^2 + 5(y+3)^2 = 15$

$$\frac{3(x-2)^2}{15} + \frac{5(y+3)^2}{15} = \frac{15}{15}$$

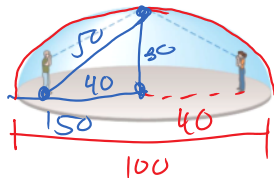5. $4x^2 + y^2 - 32x + 16y + 124 = 0$

$4x^2 - 32x + \_\_ + y^2 + 16y + \_\_ = -124$

$4(x^2 - 8x + 16) + (y^2 + 16x + \_\_ = -124 + 64 + 64$

$$\frac{4(x-4)^2}{4} + \frac{(y+8)^2}{4} = \frac{4}{4}$$

$$\frac{(x-4)^2}{1} + \frac{(y+8)^2}{4} = 1$$

SHHH! IT'S A SECRET! One interesting property of ellipses is that a wave (sound, light, etc.) emitted from one focus will be reflected from the ellipse to the exact spot of the other focus, as illustrated at the right.

50   80
40
50   40
100

For example, a "whispering gallery" is an elliptical room with an elliptical, dome-shaped ceiling. Two people standing at the foci can whisper and hear one another clearly, while individuals in other locations throughout the room cannot hear them.

PROBLEM #1 An elliptical whispering room has a height of 30 feet and a width of 100 feet.

a. Write a standard form equation of an ellipse that models the room. Assume the ellipse is centered at the origin.

$$\frac{x^2}{50^2} + \frac{y^2}{30^2} = 1$$

b. How far apart should two people in the room stand so they can whisper to each other without being heard by other people in the room?

80 ft

STATUARY HALL is a chamber in the US Capitol, and was the meeting place for the US House of Representatives from 1807 until 1857. The Hall is built to resemble an ancient European amphitheater, including a curved ceiling that is a semi-ellipse.

American folklore includes the story that John Quincy Adams, while serving in the House of Representatives, was aware of this acoustical property of ellipses, and used it to his advantage. Specifically, the tale reports that Adams positioned his desk at one focal point of the Hall, allowing him to surreptitiously eavesdrop on other House members having private conversations (or so they thought!) near the other focus.

PROBLEM #2 The elliptical ceiling in Statuary Hall is 96 feet long and 23 feet high.

a. Write the standard form equation of an ellipse that models the elliptical ceiling.

$$\frac{x^2}{48^2} + \frac{y^2}{23^2} = 1$$
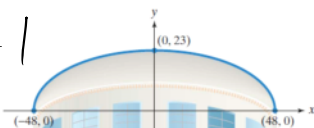
b. If John Quincy Adams placed his desk at the focus on the positive x-axis, how far from the center of Statuary Hall was it located? Round your answer to the nearest tenth of a foot.

42.13 ft

Unfortunately, the tale of John Quincy Adams deceptively using the elliptical properties of Statuary Hall to eavesdrop is untrue. ☹ The elliptical dome ceiling in the Hall was not built until 1902, long after the Hall had ceased being used by the House of Representatives. Like many other folk legends, it makes for an interesting story, and thus, has been perpetuated as an entertaining tale for tourists visiting the US Capitol.