Day 3 HW KEY

Tuesday, April 21, 2015   2:35 PM

Precalculus
Conic Section – Ellipses Day 2
Worksheet #3

Name: **Key**
Period:

Directions: Write an equation in general form for each ellipse.

1. Foci at $(\pm 4, 0)$; endpoints of major axis $(\pm 6, 0)$

major : x-axis $a = 6$
Center : $(0,0)$ $b = 2\sqrt{5}$

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

$4 = \sqrt{6^2 - b^2}$
$16 = 36 - b^2$
$b^2 = 20$
$b = \sqrt{20} = 2\sqrt{5}$

2. Foci at $(0, \pm 4)$; endpoints of major axis $(0, \pm 7)$

major : y-axis $b = 7$
center : $(0,0)$

$$\frac{x^2}{33} + \frac{y^2}{49} = 1$$

$4 = \sqrt{49 - a^2}$
$16 = 49 - a^2$
$a^2 = 33$
$a = \sqrt{33}$

3. Write the equation of the ellipse with a major axis from (-3, 5) to (9, 5) and a minor axis that is 4 units long.

major axis = 12 $b = 2$
$a = 6$ Center : $(3, 5)$

$$\frac{(x-3)^2}{36} + \frac{(y-5)^2}{4} = 1$$

$-3 + 6 = 3$
↑
x-coord.
of center

4. Write the equation of the ellipse with a center at (1, -8), a focus at (1, -2), and the end of the major axis at (1,0)

major axis: (parallel to) y-axis $-8 \to 0 = 8$
major radius
is $8 \Rightarrow b = 8$

$$\frac{(x-1)^2}{28} + \frac{(y+8)^2}{64} = 1$$

focus : $-8 \Rightarrow -2 = 6$
$6 = \sqrt{8^2 - a^2}$
$36 = 64 - a^2$
$a = \sqrt{28} = 2\sqrt{7}$

5. How does the equation of a circle relate to the equation of an ellipse?

↓   ↓

$x^2 + y^2 = r^2$ $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$
⇓

$\dfrac{x^2}{r^2} + \dfrac{y^2}{r^2} = 1$ ellipse ⇒ r is different along
horizontal & vertical
axes

circle ⇒ r is the same all the
way around

OVER!

Directions: Convert the following equations to standard form of an ellipse by completing the square. Then, graph each ellipse, including the foci.
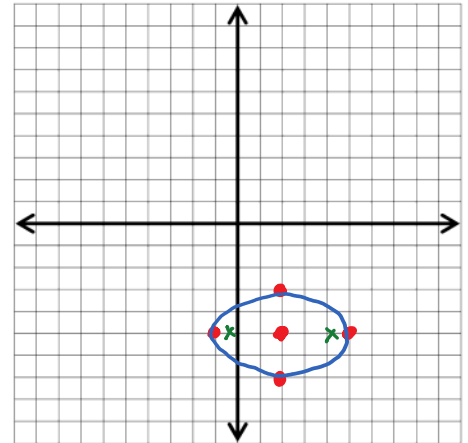
6. $4x^2 + 9y^2 - 16x + 90y + 205 = 0$

$(4x^2 - 16x) + (9y^2 + 90y) = -205$

$4(x^2 - 4x + \underline{4}) + 9(y^2 + 10y + \underline{25}) = -205 + \underline{16} + \underline{225}$

$\dfrac{4(x-2)^2}{36} + \dfrac{9(y+5)^2}{36} = \dfrac{36}{36}$

$\dfrac{(x-2)^2}{9} + \dfrac{(y+5)^2}{4} = 1$ $\underline{\text{center}: (2,-5)}$

foci : $\sqrt{3^2 - 2^2} = \sqrt{5} \Rightarrow (2-\sqrt{5}, -5)$
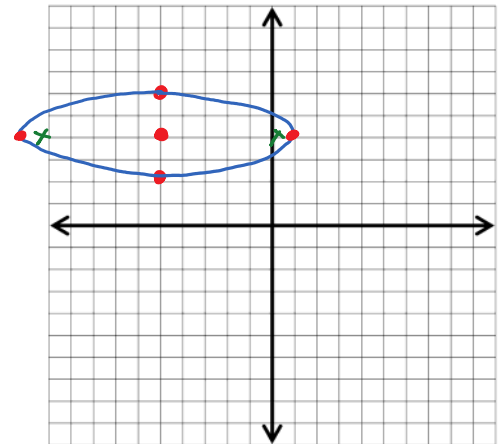$(2+\sqrt{5}, -5)$

7. $4x^2 + 36y^2 + 40x - 288y + 532 = 0$

$(4x^2 + 40x) + (36y^2 - 288y) = -532$

$4(x^2 + 10x + \underline{25}) + 36(y^2 - 8y + \underline{16}) = -532 + \underline{100} + \underline{576}$

$\dfrac{4(x+5)^2}{144} + \dfrac{36(y-4)^2}{144} = \dfrac{144}{144}$

$\dfrac{(x+5)^2}{36} + \dfrac{(y-4)^2}{4} = 1$ center : $(-5, 4)$

foci : $\sqrt{6^2 - 2^2} = \sqrt{32} = 4\sqrt{2} \Rightarrow (-5+4\sqrt{2}, 4)$
$(-5-4\sqrt{2}, 4)$

8. $49x^2 + 16y^2 + 98x - 64y - 671 = 0$

$(49x^2 + 98x) + (16y^2 - 64y) = 671$

$49(x^2 + 2x + \underline{1}) + 16(y^2 - 4y + \underline{4}) = 671 + \underline{49} + \underline{64}$

$\dfrac{49(x+1)^2}{784} + \dfrac{16(y-2)^2}{784} = \dfrac{784}{784}$

$\dfrac{(x+1)^2}{16} + \dfrac{(y-2)^2}{49} = 1$ center : $(-1, 2)$

foci : $\sqrt{7^2 - 4^2} = \sqrt{33} \Rightarrow (-1, 2+\sqrt{33})$
$(-1, 2-\sqrt{33})$