

Study Guide KEY

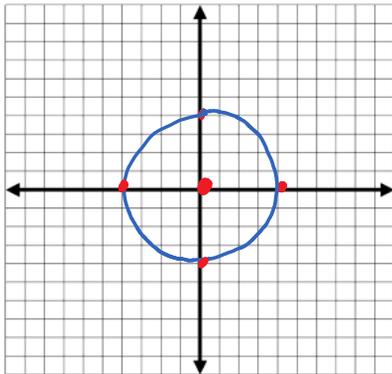
Thursday, April 30, 2015 11:37 AM

A series of horizontal blue lines for writing, with a vertical red margin line on the left side.

(#1-8) Graph each of the following. State the name of the conic and include:

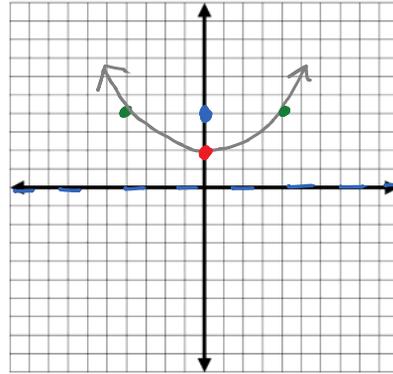
- Circles: Center and radius
- Parabolas: Vertex, focus, directrix, focal width and direction it opens
- Ellipses: Center, endpoints of major and minor axes, and foci
- Hyperbolas: Center, vertices, slopes of asymptotes and foci

1. $x^2 + y^2 = 16$



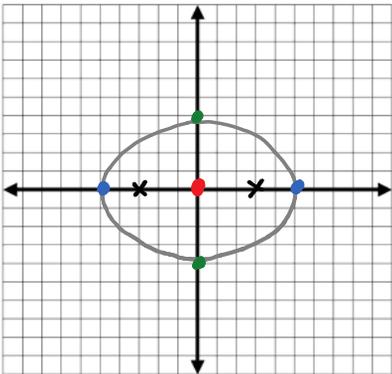
circle
 $r=4$

2. $x^2 = 8(y-2)$



parabola
vertex: $(0,2)$
Focus: $(0,4)$
dir: $y=0$
FW=8
opens up

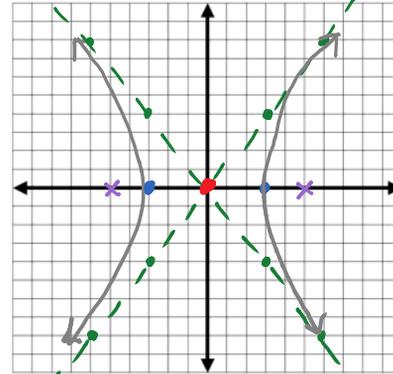
3. $\frac{x^2}{25} + \frac{y^2}{16} = 1$



ellipse
major: $(\pm 5, 0)$
minor: $(0, \pm 4)$
center: $(0, 0)$
foci: $(\pm 3, 0)$

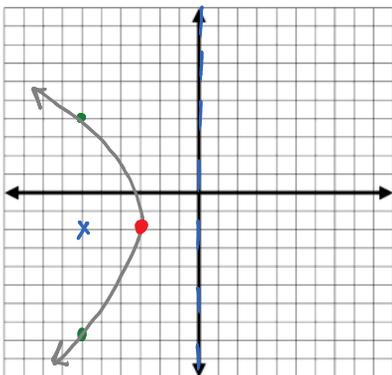
$F = \sqrt{25 - 16} = 3$

4. $\frac{x^2}{9} - \frac{y^2}{16} = 1$



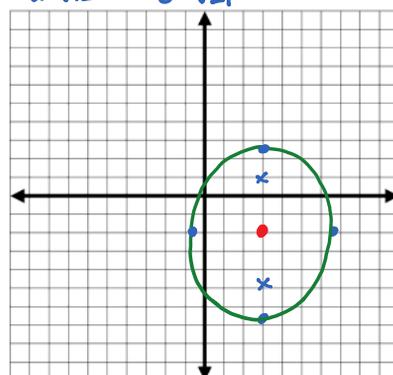
hyperbola
center: $(0, 0)$
Left / Right
vertices:
 $(\pm 3, 0)$
 $m = \pm \frac{4}{3}$
foci: $(\pm 5, 0)$
 $F = \sqrt{9 + 16} = 5$

5. $(y+2)^2 = -12(x+3)$



parabola
vertex: $(-3, -2)$
left
FW=12
focus $(-6, -2)$
 $x=0$

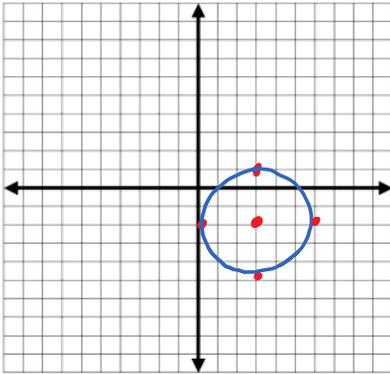
6. $\frac{(x-3)^2}{12} + \frac{(y+2)^2}{21} = 1$
 $a = \sqrt{12}$ $b = \sqrt{21}$



ELLIPSE
center: $(3, -2)$
major: $(3, -2 \pm \sqrt{21})$
minor: $(3 \pm \sqrt{3}, -2)$
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 $(3 \pm 2\sqrt{3}, -2)$
Foci: $(3, 1), (3, -5)$

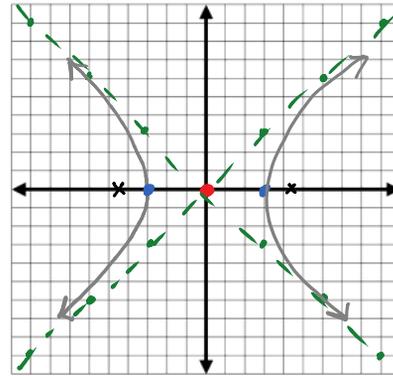
$F = \sqrt{21 - 12} = 3$

7. $(x-3)^2 + (y+2)^2 = 9$



Circle
center: (3, -2)
r = 3

8. $\frac{x^2}{9} - \frac{y^2}{9} = \frac{9}{9} \Rightarrow \frac{x^2}{9} - \frac{y^2}{9} = 1$



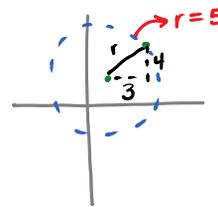
hyperbola
center: (0, 0)
Left/right
vertices: (±3, 0)
 $m = \pm \frac{3}{3} = \pm 1$
Foci: $(-3 - 3\sqrt{2}, 0)$
 $(3 + 3\sqrt{2}, 0)$
 $F = \sqrt{9+9} = \sqrt{18}$

(#9-16) For each of the following, write an equation in general form.

9. Circle; center at (-4, 1); radius = 7

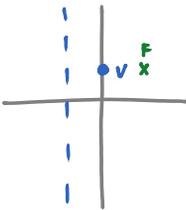
$(x+4)^2 + (y-1)^2 = 49$

10. Circle; center at (2, 5); contains (5, 9)



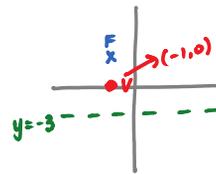
$(x-2)^2 + (y-5)^2 = 25$

11. Parabola; vertex at (0, 2); directrix $x = -2$
Right $\Rightarrow p = 2 \Rightarrow FW = 8$



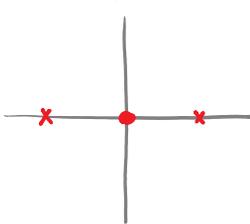
$(y-2)^2 = 8x$

12. Parabola opening up with a focus at (-1, 3) and focal width 12 $\Rightarrow p = 3$



$(x+1)^2 = 12y$

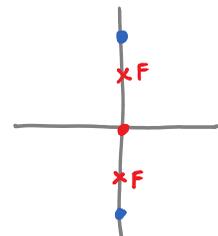
13. Ellipse; foci at (6, 0) and (-6, 0); minor axis length = 16
 $F = 6$
 $b = 8$



$F = \sqrt{a^2 - b^2}$
 $6 = \sqrt{a^2 - 64}$
 $36 = a^2 - 64$
 $a^2 = 100$

$\frac{x^2}{100} + \frac{y^2}{64} = 1$

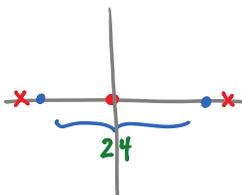
14. Ellipse; foci at (0, 3) and (0, -3); endpoints of major axis (0, 5) and (0, -5) $b = 5, f = 3$



$3 = \sqrt{25 - b^2}$
 $9 = 25 - b^2$
 $b = 4$

$\frac{x^2}{16} + \frac{y^2}{25} = 1$

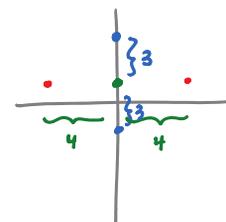
15. Hyperbola; foci at (13, 0) and (-13, 0); transverse axis length = 24 $\Rightarrow a = 12, f = 13$
Left/right



$f = \sqrt{a^2 + b^2}$
 $13 = \sqrt{12^2 + b^2}$
 $b = 5$

$\frac{x^2}{144} - \frac{y^2}{25} = 1$

16. Hyperbola; vertices (4, 1) and (-4, 1); ends of conjugate axis at (0, 4) and (0, -2) Left/right



$a = 4$
 $b = 3$ center: (0, 1)

$\frac{x^2}{16} - \frac{(y-1)^2}{9} = 1$

(#17-22) Identify the shape and convert the following equations to general form by completing the square, if necessary.

17. $x^2 - 6x + y^2 - 8y = 0$

$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 0 + 9 + 16$$

$$(x-3)^2 + (y-4)^2 = 25$$

circle centered @ (3,4) w/ r=5

18. $\frac{4y^2}{36} - \frac{9x^2}{36} = \frac{36}{36}$

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

hyperbola centered @ origin opening up/down

19. $y^2 + 4y - 4x = 0$

$$y^2 + 4y + 4 = 4x + 4$$

$$(y+2)^2 = 4(x+1)$$

parabola opening right w/ a vertex @ (-1,-2)

20. $2x^2 + 3y^2 + 4x - 12y = 4$

$$2x^2 + 4x + 3y^2 - 12y = 4$$

$$2(x^2 + 2x + 1) + 3(y^2 - 4y + 4) = 4 + 2 + 12$$

$$\frac{2(x+1)^2}{18} + \frac{3(y-2)^2}{18} = \frac{18}{18}$$

$$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{6} = 1$$

ELLIPSE centered @ (-1,2) w/ main axis parallel to x-axis

21. $4x^2 - y^2 - 32x + 16y - 128 = 0$

$$4x^2 - 32x - y^2 + 16y = 128$$

$$4(x^2 - 8x + 16) - (y^2 - 16y + 64) = 128 + 64 - 64$$

$$\frac{4(x-4)^2}{128} - \frac{(y-8)^2}{128} = \frac{128}{128}$$

$$\frac{(x-4)^2}{32} - \frac{(y-8)^2}{128} = 1$$

hyperbola centered @ (4,8) opening left/right

22. $4y = \left[\frac{1}{4}x^2 + 1x - 4 \right] 4$

$$4y = x^2 + 4x - 16$$

$$4 + 4y + 16 = x^2 + 4x + 4$$

$$4(y+5) = (x+2)^2$$

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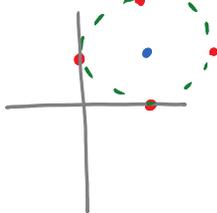
$$(x+2)^2 = 4(y+5)$$

parabola opening up w/ a vertex @ (-2,-5)

(#23-24) Write parametric equations for each of the following.

23. A circle that is tangent to the x-axis at (5,0) and the y-axis at (0,5).

r=5, center @ (5,5)



$$x = 5 + 5\cos t$$

$$y = 5 + 5\sin t$$

24. A hyperbola with standard equation

$$\frac{(y+2)^2}{9} - \frac{(x-3)^2}{16} = 1$$

center: (3,-2) up/down

$$x = 3 + 4\tan t$$

$$y = -2 + 3\sec t$$

(#25-26) Eliminate the parameter and identify the name of the conic section.

25. $\left. \begin{matrix} x = 2 + 4\cos t \\ y = 3 + 3\sin t \end{matrix} \right\}$ ellipse!
centered: (2,3)
a=4
b=3

$$\frac{(x-2)^2}{16} + \frac{(y-3)^2}{9} = 1$$

26. $\left. \begin{matrix} x = 4\sec t + 2 \\ y = 3\tan t + 3 \end{matrix} \right\}$ hyperbola opening left/right centered (2,3)
a=4
b=3

$$\frac{(x-2)^2}{16} - \frac{(y-3)^2}{9} = 1$$