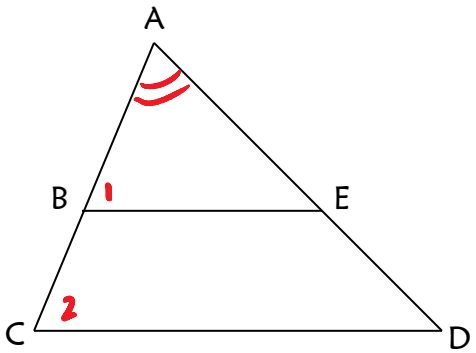
8.5 Three Theorems Involving Proportions

Side Splitter Theorem: Given: $\overline{BE} \parallel \overline{CD}$ Prove: $\frac{BC}{AB} = \frac{ED}{AE}$



1. $\angle 1 \cong \angle 2$ — 1. If // lines → corr. ∠'s ≅
2. $\angle A \cong \angle A$ — 2 Reflexive prop
3. $\triangle ABE \sim \triangle ACD$ — 3. AA~
4. $\frac{AC}{AB} = \frac{AD}{AE}$ — 4. CSSTP
5. $AC = AB + BC$, $AD = AE + ED$ — 5. Addition
6. $\frac{AB+BC}{AB} = \frac{AE+ED}{AE}$ — 6. Substitution
7. $\frac{AB}{AB} + \frac{BC}{AB} = \frac{AE}{AE} + \frac{ED}{AE}$
8. $1 + \frac{BC}{AB} = 1 + \frac{ED}{AE}$
9. $\frac{BC}{AB} = \frac{ED}{AE}$

Side-Splitter Theorem (pg 351):

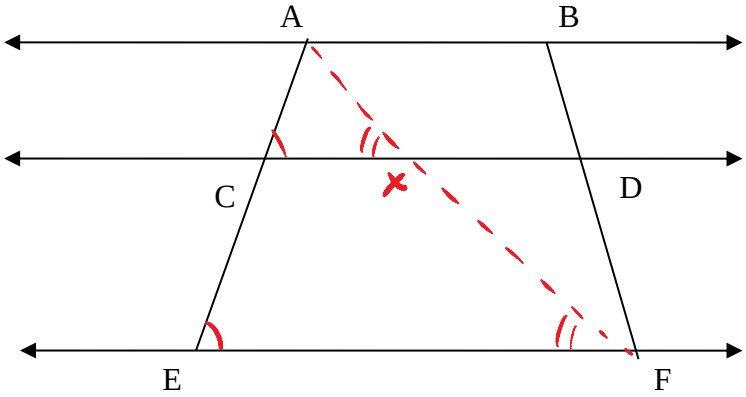
$$\frac{AB}{BC} = \frac{AE}{ED}$$

Side Splitter Corollary:

Given the three parallel lines and transversal below:

Given: $\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$

Prove: $\frac{AC}{CE} = \frac{BD}{DF}$



$\frac{AC}{CE} = \frac{AX}{XF}$ $\frac{AX}{XF} = \frac{BD}{DF}$

$\frac{AC}{CE} = \frac{BD}{DF}$

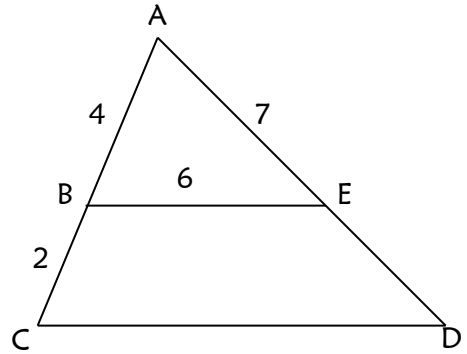
Substitution

Side-Splitter Corollary (pg 351):

$$\frac{AC}{CE} = \frac{BD}{DF}$$

A few quick examples

Ex: 1 Given: $\overline{BE} \parallel \overline{CD}$
Find: ED and CD



$\frac{4}{2} = \frac{7}{ED}$

$4ED = 14$

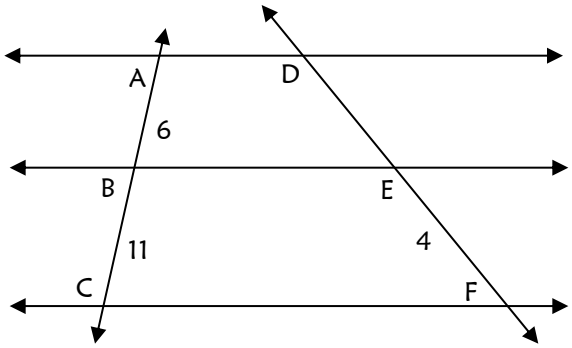
$\boxed{ED = 7/2}$

$\frac{4}{6} = \frac{6}{CD}$

$4CD = 36$

$\boxed{CD = 9}$

Ex: 2 Given: $\overline{AD} \parallel \overline{BE} \parallel \overline{CF}$
Find: DE



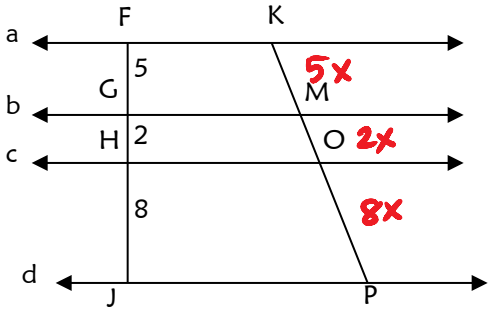
$\frac{6}{11} = \frac{DE}{4}$

$11DE = 24$

$\boxed{DE = 24/11}$

Ex: 3 Given a ∥ b ∥ c ∥ d and KP = 36, find KM.

Find: KM, MO, and OP



$5x + 2x + 8x = 36$

$15x = 36$

$x = \frac{12}{5}$

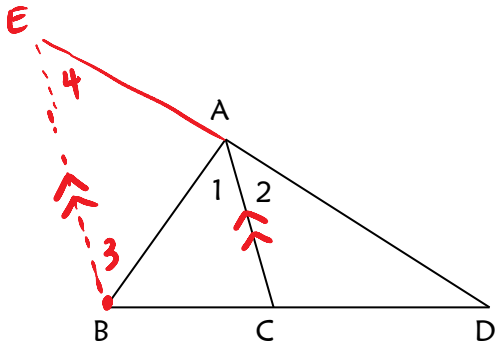
$KM = 5\left(\frac{12}{5}\right) = 12$

$MO = 2\left(\frac{12}{5}\right) = \frac{24}{5}$

$OP = 8\left(\frac{12}{5}\right) = \frac{96}{5}$

Angle Bisector Theorem:

Given: $\overrightarrow{AC}$ bisects ∠BAD



$\angle 1 \cong \angle 2$ (def bis)

$\angle 1 \cong \angle 3$ (alt int)

$\angle 2 \cong \angle 4$ (corr.)

$\angle 3 \cong \angle 4$ (transitive)

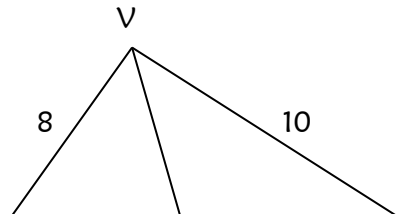
$\overline{AE} \cong \overline{AB}$ If △ → ⊿

$\frac{DA}{AE} = \frac{DC}{CB}$ (side splitter)

$\frac{DA}{AB} = \frac{DC}{CB}$ (Substitution)

Angle Bisector Theorem (pg 352):

$\frac{AB}{BC} = \frac{AD}{CD}$



Ex. 4 Given: $\angle RVS \cong \angle SVT$
Lengths as shown

Find: ST

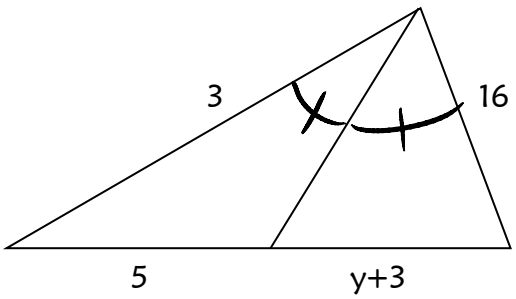
$\frac{8}{4} = \frac{10}{x}$

$\frac{2}{1} = \frac{10}{x}$

$2x = 10$

$\boxed{x = 5}$

Ex. 5 Find y:



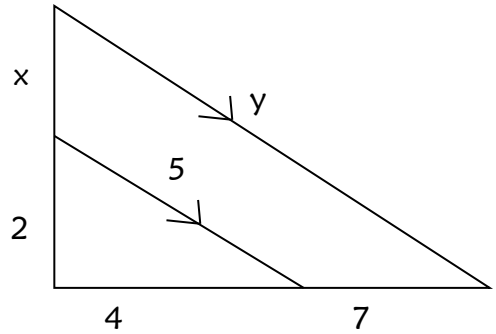
$\frac{3}{5} = \frac{16}{y+3}$

$3y + 9 = 80$

$3y = 71$

$\boxed{y = \frac{71}{3}}$

Ex. 6 Solve for x and y:



$\frac{2}{x} = \frac{4}{7}$

$4x = 14$

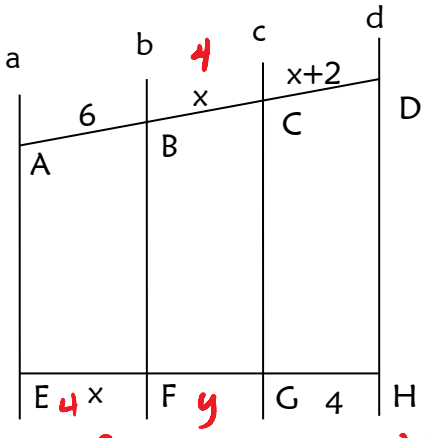
$\boxed{x = 7/2}$

$\frac{4}{11} = \frac{5}{y}$

$4y = 55$

$\boxed{y = 55/4}$

Ex. 7
Given: a||b||c||d
Find: AC



$\frac{6}{x} = \frac{x+2}{4}$

$x^2 + 2x = 24$

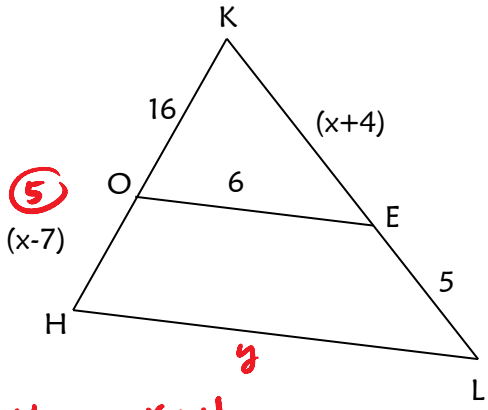
$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = \cancel{-6}, 4$

$\boxed{AC = 10}$

Ex. 8 Find the perimeter of triangle KHL



$\frac{16}{x-7} = \frac{x+4}{5}$

$x^2 - 3x - 28 = 80$

$x^2 - 3x - 108 = 0$

$(x-12)(x+9) = 0$

$\boxed{x = 12}, \cancel{-9}$

$\frac{16}{21} = \frac{6}{y}$

$16y = 126$

$\boxed{y = \frac{63}{8}}$