8.1 NOTES – Relationships Involving Polygons

Consider the proportion $\frac{a}{b}=\frac{c}{d}$ terms: $\frac{1st}{2nd}=\frac{3rd}{4th}$

The 1st and 4th terms are called <u>extremes</u>.
The 2nd and 3rd terms are called <u>means</u>.

Mean -Extremes Products Theorem

If $\frac{a}{b}=\frac{c}{d}$, then $ad=bc$.

Means-Extremes Ratio Theorem If $pq=rs$, then $\frac{p}{r}=\frac{s}{q}$, $\frac{p}{s}=\frac{r}{q}$, and $\frac{r}{p}=\frac{q}{s}$.

Example 1: $\frac{2}{x}=\frac{10}{14}$, solve for x.

Example 2: If $3x=4y$, find the ratio of x to y.

Example 3: If $\frac{4}{2x-3y}=\frac{5}{x+2y}$ Find the ratio of x to y.

Example 4: Find the fourth term of a proportion if the 1st three terms are 2, 3, 4.

<u>Geometric Mean/Mean Proportional</u> – Means in a proportion are equal *examples:*
<u>Arithmetic Mean</u> – Average of 2 numbers

Example 5: Find the geometric mean between 3 and 27.

Example 6: Find the mean proportional between 4 and 16.

Example 7: Find the arithmetic mean between 5 and 9

Example 8: Show that $\frac{a}{b}=\frac{c}{d}$ is equivalent to $\frac{a+b}{b}=\frac{c+d}{d}$

Partner Practice 

1. Find the arithmetic mean between 6 and 30.

$$\frac{6+30}{2} = \boxed{18}$$

2. Find the fourth proportional of 6, 8, and 9.

$$\frac{6}{8} = \frac{9}{x} \qquad 6x = 72 \qquad \boxed{x = 12}$$

3. Find the geometric mean between 16 and 25.

$$\frac{16}{x} = \frac{x}{25} \qquad x^2 = 400 \qquad \boxed{x = \pm 20}$$

4. Find the mean proportional between 10 and 8.

$$\frac{10}{x} = \frac{x}{8} \qquad x^2 = 80$$

$$x^2 = \pm\sqrt{80} \text{ or } \boxed{\pm 4\sqrt{5}}$$

5. Given: 3(x + 2y + 6) = 2(5x + y + 9)
   Find the ratio of x to y:

$$3x + 6y + \cancel{18} = 10x + 2y + \cancel{18}$$

$$\frac{4\cancel{y}}{7y} = \frac{\cancel{7}x}{\cancel{7}y}$$

$$\boxed{\frac{x}{y} = \frac{4}{7}}$$

6. Solve for x: $\dfrac{x+5}{6} = \dfrac{7}{x-6}$

$$(x+5)(x-6) = 42$$

$$x^2 - 6x + 5x - 30 = 42$$

$$x^2 - x - 72 = 0$$

$$(x-9)(x+8) = 0$$

$$\boxed{x = 9, -8}$$

7. $3\sqrt{6}$ is the mean proportional between 6 and what number?

$$\frac{6}{3\sqrt{6}} = \frac{3\sqrt{6}}{x} \qquad 6x = 9(6) \qquad \boxed{x = 9}$$

8. Find the ratio of x to y. gx – fy = hx + my

$$gx - hx = my + fy$$

$$\frac{x\cancel{(g-h)}}{y\cancel{(g-h)}} = \frac{\cancel{y}(m+f)}{\cancel{y}(g-h)}$$

$$\boxed{\frac{x}{y} = \frac{m+f}{g-h}}$$