7.2 DAY 3: MORE MATRICES

① identity
② inverses
③ determinant

CALCULATOR INSTRUCTIONS

The Identity Matrix: a square matrix that has 1's on the main diagonal & 0's elsewhere

examples: $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$

the identity matrix will help us prove when 2 matrices are inverses of one another

Inverse of a square matrix: two square matrices, matrix A & matrix B, are inverses of one another if their product is an identity matrix

$$AB = BA = I$$

If they are inverses, we denote $B = A^{-1}$ or $A = B^{-1}$

example 1: Prove that $A = \begin{bmatrix}3&-2\\-1&1\end{bmatrix}$ & $B = \begin{bmatrix}1&2\\1&3\end{bmatrix}$ are inverse matrices.

$$AB = \begin{bmatrix}3&-2\\-1&1\end{bmatrix}\begin{bmatrix}1&2\\1&3\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix} \checkmark$$

use calculator

$$BA = \begin{bmatrix}1&2\\1&3\end{bmatrix}\begin{bmatrix}3&-2\\-1&1\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix} \checkmark$$

$\therefore B = A^{-1}$ & $A = B^{-1}$

*when a square matrix has an inverse, we call it <u>nonsingular</u>

*if a square matrix has no inverse, we call it <u>singular</u>

You try! Are the following 2 matrices inverses?

$A = \begin{bmatrix}-2&1&3\\1&2&-2\\0&1&-1\end{bmatrix}$ $B = \begin{bmatrix}0&1&-2\\0.25&0.5&-0.25\\0.25&0.5&-1.25\end{bmatrix}$

*use calculator

$$AB = \begin{bmatrix}-2&1&3\\1&2&-2\\0&1&-1\end{bmatrix}\begin{bmatrix}0&1&-2\\0.25&0.5&-0.25\\0.25&0.5&-1.25\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \checkmark$$

$\boxed{\text{YES!}}$

$$BA = \begin{bmatrix}0&1&-2\\.25&.5&-.25\\.25&.5&-1.25\end{bmatrix}\begin{bmatrix}-2&1&3\\1&2&-2\\0&1&-1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \checkmark$$

Determinant of a Square Matrix

the determinant tells us if a square matrix has an inverse the determinant tells us if a square matrix has an inverse

2×2 MATRICES

$$\boxed{\det A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc}$$

If the $\det A \neq 0$, then you can find the inverse from the following formula

$$\boxed{\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

example 1: does the following matrix have an inverse? if so, find it!

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

$\det A = 3(2) - 1(4) = 2 \Rightarrow$ so A has an inverse

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -0.5 \\ -2 & 1.5 \end{bmatrix}$$

check with calculator (det & A^{-1})

you try: $B = \begin{bmatrix} -1 & -7 \\ 3 & 20 \end{bmatrix}$

$\det B = (-1)(20) - (-7)(3) = 1 \Rightarrow$ so B has an inverse

$$B^{-1} = \frac{1}{1}\begin{bmatrix} 20 & 7 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} 20 & 7 \\ -3 & -1 \end{bmatrix}$$

OTHER SQUARE MATRICES

*use your calculator to find the value of the determinant. if $\det \neq 0$, then use the inverse button to find the inverse matrix

example: find C^{-1} if it exists. $C = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 0 & 1 \end{bmatrix}$

$\det C = -10$

$$C^{-1} = \begin{bmatrix} 0.1 & 0.2 & -0.5 \\ 0.5 & 0 & 0.5 \\ 0.1 & 0.2 & 0.5 \end{bmatrix}$$