Day 17 Notes

Tuesday, March 17, 2015 9:27 AM

Section 6.6 Day 2 Notes

DeMoivre's Theorem

Recall from yesterday:

Finding the Product of 2 Complex Numbers

$r_1 \cdot r_2 \ Cis(\theta_1 + \theta_2)$

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$.

Then $z_1 \cdot z_2 =$ $r_1 \cdot r_2 [\cos(\theta_1+\theta_2) + i \sin(\theta_1+\theta_2)]$

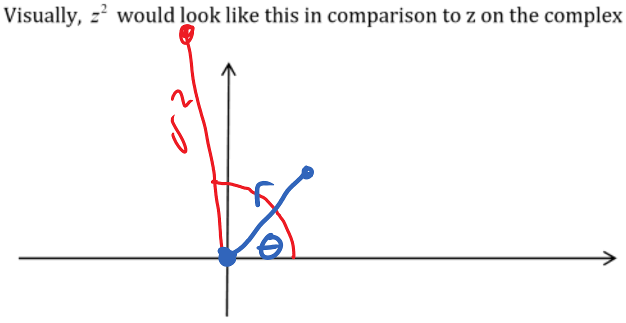
$z = r\,Cis\,\theta$

So what would z^2 look like in trig form?

$z \cdot z = r \cdot r \ Cis(\theta + \theta)$

$r^2 \ Cis\,(2\theta)$

Visually, z^2 would look like this in comparison to z on the complex plane.



What about z^3?

$z^2 \cdot z = r^2 \ Cis\,2\theta \cdot r \ Cis\,\theta$

$= r^3 \ Cis(\ 3\theta\ )$

Can you make a prediction about z^4?

$r^4 \ Cis\,4\theta$

DeMoivre's Theorem

Let $z = r(\cos\theta + i\sin\theta)$ and let n be a positive integer. Then:

$z^n =$ r^ Cis(nθ)

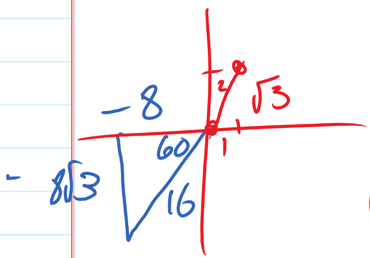
* only works w/ trig Form!

Step 1: Change to trig Form

Step 2: Apply DeMoivre's

Step 3: Change back to standard Form

1. Find $\left(1+i\sqrt{3}\right)^4$ using DeMoivre's Theorem.



r = 2

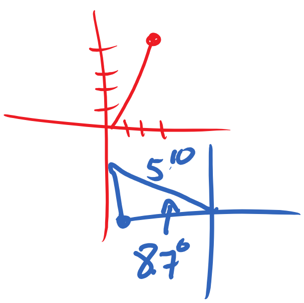
θ = 60

$(2\,\text{Cis}\,60)^4 = 2^4\,\text{Cis}(4\cdot 60)$

$16\ \text{Cis}\ (240)$

$\boxed{-8 - 8i\sqrt{3}}$

2. Find $(3+4i)^{10}$.

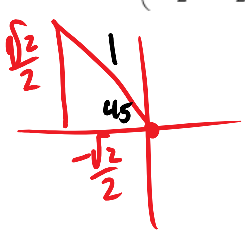


① $(5\ \text{Cis}\,53.13°)^{10}$

② $5^{10}\,\text{Cis}(10\cdot 53.13)$

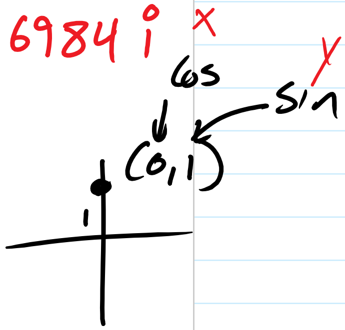
③ $-9653287 + 1476984i$

3. Find $\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^6$.



① $(1\,\text{Cis}(135))^6$

② $1^6\ \text{Cis}(6\cdot 135)$



$\frac{\sqrt{2}}{2}\cdot\sqrt{2} = 1$

③ $1(0+1i)$

$\boxed{i} = \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^6$