Day 16 Notes

Monday, March 16, 2015 2:27 PM

Section 6.6 Day 1

Complex Plane and Trig. Form

How do we graph real numbers? rectangular or Polar

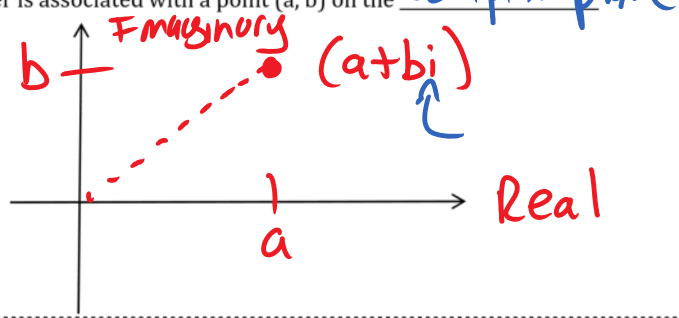
How do we graph complex numbers? Complex plane

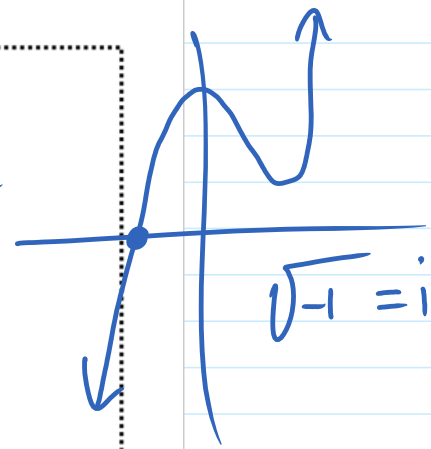
Real numbers are placed along the x-axis (Horizontal)

Imaginary numbers are placed along the y-axis (Vertical)

Thus, each complex number is associated with a point (a, b) on the Complex plane

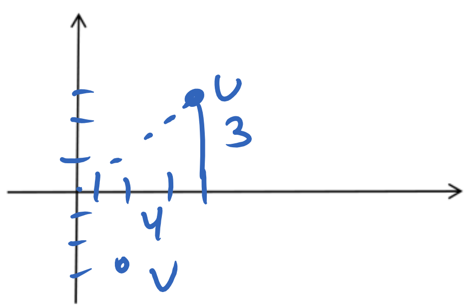
a+bi





$\sqrt{-1} = i$

Example: Plot u = 4 + 3i and v = 2 – 3i on the complex plane.



How could we find the distance the point "u" is from the origin in the last example?

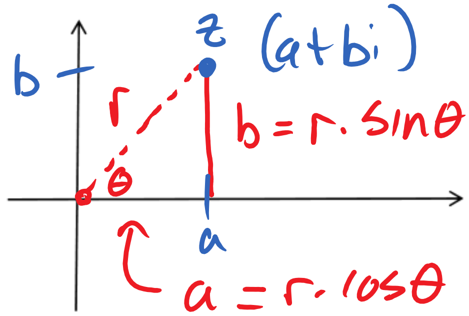
$$\sqrt{u^2} = \sqrt{4^2+3^2} \qquad u = 5$$

Definition: The absolute value or modulus of a complex number u = a + bi is $|u| = \sqrt{a^2+b^2}$. Where $|a+bi|$ is the distance a + bi is from the origin.

Magnitude!

TRIGONOMETRIC FORM OF COMPLEX NUMBERS

standard Form: $a+bi$



So a = $r \cdot \cos\theta$ and b = $r \cdot \sin\theta$ and z = a + bi.

So through substitution:

z = $r \cdot \cos\theta + r\sin\theta i$ = $r(\cos\theta + \sin\theta\, i)$

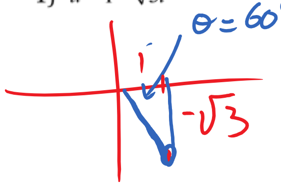
This is called the trig Form of z = a + bi.

REMINDER:

- $r = \sqrt{a^2+b^2}$ $\quad\theta = \tan^{-1}\left(\frac{b}{a}\right)$
- r ⇒ modulus (magnitude)
- θ ⇒ argument (direction)

EXAMPLE: Find the trigonometric form with $0 \le \theta < 2\pi$ for the complex number.

1) $u = 1 - \sqrt{3}i$

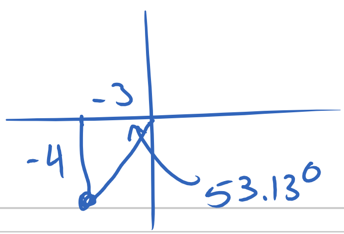


$r = 2$

$\theta = 300°$

$u = 2(\cos 300 + \sin 300 i)$

2) $v = -3 - 4i$



$r = 5$

$\theta = 233.13°$

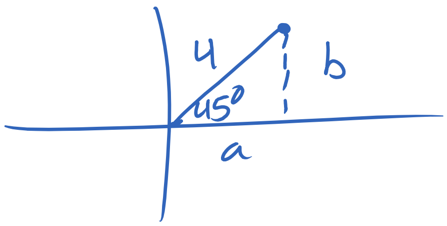
$u = 5(\cos 233.13 + \sin 233.13 i)$

TRY: $w = -4 + 2i$

EXAMPLE: Write the complex number in standard form a + bi.

Example: Write the complex number in standard form a + bi.

1) $4(\cos 45° + i\sin 45°)$



$\boxed{2\sqrt{2} + 2\sqrt{2}\, i}$

2) $10(\cos 150° + i\sin 150°)$

Finding the Product of 2 Complex Numbers

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$.

Then $z_1 \cdot z_2 = r_1 \cdot r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$

Example: Multiply z_1 and z_2 and write in standard form.

1) $z_1 = 25\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$, $z_2 = 14\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$

2) $z_1 = 7(\cos 33° + i\sin 33°)$, $z_2 = 3(\cos 121° + i\sin 121°)$

Finding the quotient of 2 complex numbers

$\frac{z_1}{z_2} =$

Example: $z_1 = 2\sqrt{3}\left(\cos 135° + i\sin 135°\right), z_2 = 6\sqrt{3}\left(\cos 30° + i\sin 30°\right)$