Precalculus   Section 5.6 – The Law of Cosines

The Law of Sines can be used to solve triangles when you know $\underline{ASA \quad AAS \quad {}^*SSA}$.
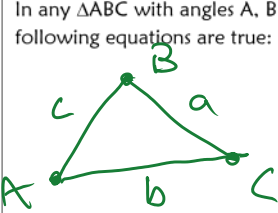
The Law of Cosines can be used to solve triangles when you know $\underline{SSS \quad SAS}$.

(Either rule can be used for $\underline{SSA}$, but remember that there could be 0, 1, or 2 triangles – we'll deal with that later.)

The Law of Cosines is called the "generalized Pythagorean Theorem."

> The Law of Cosines states:
>
> In any ΔABC with angles A, B, and C opposite sides a, b, and c, respectively, the following equations are true:
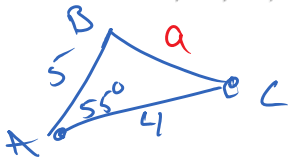>
> $$a^2 = b^2 + c^2 - 2bc \cos A$$
> $$b^2 = a^2 + c^2 - 2ac \cos B$$
> $$c^2 = a^2 + b^2 - 2ab \cos C$$

$8 - 2x$

Examples: Find the missing side.
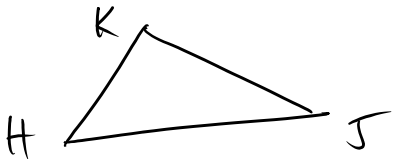1. $\triangle ABC, b=4, c=5, m\angle A = 55°$

$$a^2 = 4^2 + 5^2 - 2(4)(5) \cos 55$$
$$= 16 + 25 - 40 \cos 55$$
$$= 41 - 40 \cos 55$$

$$a \approx 4.25$$

2. $\triangle HJK, h=8, j=6, m\angle K = 172°$

$$k^2 = 8^2 + 6^2 - 2(8)(6) \cos 172°$$
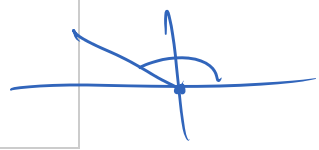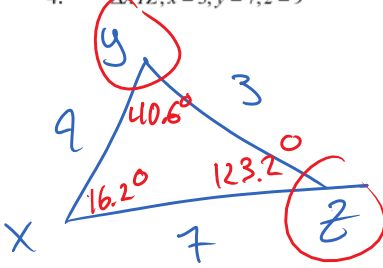$$= 100 - 96 \cos 172$$

$$k \approx 13.97$$

$\sqrt{ANS}$

Try it! Find the missing side.
3. $\triangle KSD, m\angle S = 127°, k=16, d=3$

Find the angles of the triangle.

4. $\triangle XYZ, x = 3, y = 7, z = 9$

$$7^2 = 3^2 + 9^2 - 2(3)(9)\cos Y$$

$$49 = 90 - 54\cos Y$$
$$\underline{-90 \quad -90}$$

$$-41 = -54\cos Y$$

$$\frac{41}{54} = \cos Y$$

$$Y = \cos^{-1}\left(\frac{41}{54}\right)$$

$$9^2 = 7^2 + 3^2 - 2(7)(3)\cos Z$$
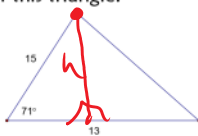
$$23 = -42\cos Z$$

$$\frac{23}{-42} = \cos Z$$

Try it! Find the angles of the triangle.

5. $\triangle AUG, a = 5, u = 8, g = 10$

Area of a Triangle – 2 Formulas

Area of a Triangle = $\underline{\frac{1}{2}Bh}$

Find the area of this triangle:

$$\sin 71 = \frac{h}{15}$$

Area of a Triangle

Find missing pieces / height

This formula is for when you know ___AAS, SAS___

If you know ___**SSS**___ instead, you can use this formula from Geometry:

✝SAS ✝ASA

Heron's Formula

$S = $ semi perimeter $= \frac{1}{2}P$

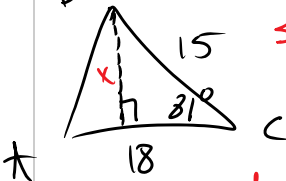$$A_\triangle = \sqrt{s(s-a)(s-b)(s-c)}$$

This formula is for when you know _____

$\frac{1}{2}b \cdot h$

Examples: Find the area of the given triangle to the nearest 10th.

6. $\triangle ABC, b=18, a=15, m\angle C=81°$ **SAS**
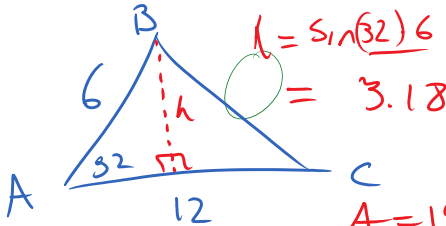

$\sin 81 = \frac{x}{15}$

$x = 14.82$

$A = \frac{1}{2}(14.82)(18) = \boxed{133.34}$

7. $\triangle CAT, c=4, a=6, t=7$

$S = 8.5$

$A = \sqrt{8.5(4.5)(2.5)(1.5)}$

Try it! Find the area of the given triangle to the nearest 10th.

8. $\triangle ABC, c=6, b=12, m\angle A=32°$

$h = \sin(32)\,6$

$= 3.18$

$A = 19.07$

9. $\triangle HPT, h=5, p=7, t=11$

$S = 11.5$

$\boxed{\sqrt{11.5(.5)(4.5)(6.5)}}$

≈ 12.97

10. $\triangle DOG, d=6, m\angle O=66°, m\angle G=29°$ (hint: how can you find the side you need first?)