5.5 Day 2 KEY

Saturday, November 1, 2014 9:47 AM

When solving triangles . . .

① State the case – can you use Law of Sines?
(need a pair → or can I easily find one?)

② if you have an ASA or AAS. if you have a SSA case

⇓ exactly one triangle

⇓ 0 triangles
1 triangle
2 triangles
AMBIGUOUS CASE

What does the ambiguous case mean??

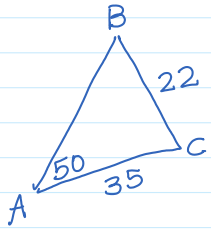
We don't know how many triangles we have . . . 0, 1, 2

*In SSA cases . . . always assume you have 2 triangles unless you prove otherwise

But how do we know?? Let's try a few examples . . .

State the case. Then solve the triangle(s).

① In $\triangle ABC$, $A = 50°$, $a = 22$, $b = 35$.



SSA ⇒ ambiguous case

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 50}{22} = \frac{\sin B}{35}$$

$$\frac{22\sin B}{22} = \frac{35\sin 50}{22}$$

$$\sin B = \frac{35\sin 50}{22}$$

$$\sin B = 1.22$$

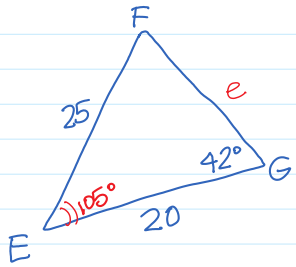
$$B = \sin^{-1}(1.22)$$

B doesn't exist

no triangle

② In $\triangle EFG$, $G = 42°$, $g = 25$, $f = 20$.

SSA ⇒ ambiguous case!



$\frac{\sin G}{g} = \frac{\sin F}{f}$

$\frac{\sin 42}{25} = \frac{\sin F}{20}$

$\frac{25\sin F}{25} = \frac{20\sin 42}{25}$

$\sin F = \frac{20\sin 42}{25}$

$\sin F = .54$

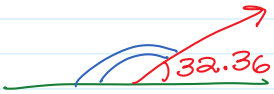
$F = \sin^{-1}(.54)$

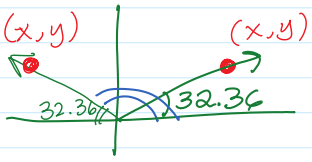
$F = 32.36°$

↑ at least one Δ

Supplement of F = 147.64°

147.64° + 42 ↑ greater than 180° (no 2nd Δ)





$E = 180 - (32.36 + 42)$

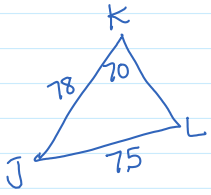
$E = 105°$

$\frac{\sin E}{e} = \frac{\sin G}{g}$

$\frac{\sin 105}{e} = \frac{\sin 42}{25}$

$e = \frac{25\sin 105}{\sin 42} = 36.1$

③ In ΔJKL, K = 70°, k = 75, l = 78. SSA



$\frac{\sin K}{k} = \frac{\sin L}{l}$

$\frac{\sin 70}{75} = \frac{\sin L}{78}$

$\sin L = \frac{78\sin 70}{75}$

$L = \sin^{-1}(.98)$

$L = 77.76°$

(at least 1 Δ)

2nd Δ?

Supplement of ∠L + ∠K ? < 180

$(180 - 77.76) + 70 \overset{?}{<} 180$

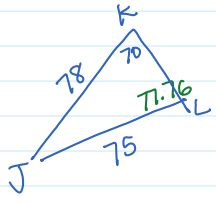
$102.24 + 70 < 180$ ✓ yes

↓ room for a 3rd ∠

2 triangles

1st triangle

L = 77.76°



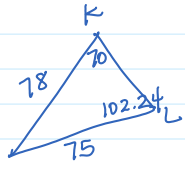
$J = 180 - (70 + 77.76) = 32.24°$

$\frac{\sin 32.24}{j} = \frac{\sin 77.76}{78}$

$j = 42.58$

2nd triangle

L = 102.24°



$J = 180 - (70 + 102.24)$

$= 7.76°$

$\frac{\sin 7.76}{j} = \frac{\sin 102.24}{78}$

$j = 10.78$

THE STEPS:

① use law of sines to complete your pair & find the missing angle opposite your given side)

② does that angle exist?

no! ⇓

yes! ⇓

you have at least one Δ. Check for 2...

③ draw that angle as a reference angle in quad 2 ⇒ find the obtuse angle to complete it

④ add the obtuse ∡ to the other given ∡.

⇓ sum > 180

⇓ sum < 180

<u>You try!</u> State the case & solve the triangle(s)...

In ΔABC, $A = 40°$, $a = 13$, $b = 16$.