5.4B HW

Tuesday, February 17, 2015 5:46 PM

- **P. 475: 6, 8, 10, 17–20, 24, 27, 31–36**

(6) $\sin 2x = \sin x$
$\sin 2x - \sin x = 0$
$2\sin x\cos x - \sin x = 0$
$\sin x(2\cos x - 1) = 0$
$\sin x = 0$: $x = 0, \pi$
$2\cos x - 1 = 0$: $\cos x = 1/2$: $x = \frac{\pi}{3}, \frac{5\pi}{3}$

(8) $\cos 2x = \cos x$
$\cos 2x - \cos x = 0$
$2\cos^2 x - 1 - \cos x = 0$
$2\cos^2 x - \cos x - 1 = 0$
$(2\cos x + 1)(\cos x - 1) = 0$
$\cos x = -1/2$: $x = \frac{2\pi}{3}, \frac{4\pi}{3}$
$\cos x = 1$: $x = 0$

(17) $2\csc 2x = \csc^2 x \tan x$ ✓
$2\left(\frac{1}{\sin 2x}\right)$
$\frac{2}{2\sin x\cos x}$
$\frac{1}{\sin x\cos x} \cdot \frac{\sin x}{\sin x} = \frac{1}{\sin^2 x} \cdot \frac{\sin x}{\cos x}$
$= \csc^2 x \tan x$ ✓

(18) $2\cot 2x = \cot x - \tan x$
$2\left(\frac{1}{\tan 2x}\right) = \cot x - \tan x$ ✓
$\frac{2}{\frac{2\tan x}{1-\tan^2 x}} = 2 \cdot \frac{1-\tan^2 x}{2\tan x}$
$= \frac{1}{\tan x} - \frac{\tan^2 x}{\tan x}$
$= \cot x - \tan x$ ✓

(19) $\sin 3x = (\sin x)(4\cos^2 x - 1)$ ✓
$\sin(2x + x)$
$\sin 2x\cos x + \cos 2x \sin x$
$(2\sin x\cos x)\cos x + (2\cos^2 x - 1)\sin x$
$2\sin x\cos^2 x + 2\sin x\cos^2 x - \sin x$
$4\sin x\cos^2 x - \sin x$
$(\sin x)(4\cos^2 x - 1)$ ✓

(20) $\sin 3x = \sin x(3 - 4\sin^2 x)$ ✓
$\sin(2x + x)$
$\sin 2x\cos x + \cos 2x \sin x$
$(2\sin x\cos x)\cos x + (1 - 2\sin^2 x)\sin x$
$2\sin x\cos^2 x + \sin x - 2\sin^3 x$
$\sin x(2\cos^2 x + 1 - 2\sin^2 x)$
$\sin x[2(1 - \sin^2 x) + 1 - 2\sin^2 x]$
$\sin x[2 - 2\sin^2 x + 1 - 2\sin^2 x]$
$\sin x(3 - 4\sin^2 x)$ ✓

(24) $\cos 2x + \sin x = 0$
$1 - 2\sin^2 x + \sin x = 0$
$-2\sin^2 x + \sin x + 1 = 0$
$-(2\sin^2 x - \sin x - 1) = 0$
$-(2\sin x + 1)(\sin x - 1) = 0$
$\sin x = -\frac{1}{2}$: $x = \frac{7\pi}{6}, \frac{11\pi}{6}$
$\sin x = 1$: $x = \frac{\pi}{2}$

(31) $\sin\left(\frac{30}{2}\right) = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$

(32) $\tan\left(\frac{390}{2}\right) = \tan\left(\frac{30}{2}\right) = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}}$
$= 2 - \sqrt{3}$

(33) $\cos\left(\frac{150}{2}\right) = \sqrt{\frac{1 + (-\sqrt{3}/2)}{2}}$
$= \frac{\sqrt{2 - \sqrt{3}}}{2}$

(34) $\sin\left(\frac{5\pi}{6}\right) = \sqrt{\frac{1 + \sqrt{3}/2}{\quad}} = \sqrt{2 + \sqrt{3}}$

(35) $\tan\left(\frac{7\pi}{6} \div 2\right) = \frac{1 + \frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -2 - \sqrt{3}$

(34) $\sin\left(\frac{\frac{5\pi}{6}}{2}\right) = \sqrt{\frac{1+\sqrt{3}/2}{2}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$

(35) $\tan\left(\frac{\frac{7\pi}{6}}{2}\right) = \frac{1+\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \boxed{-2-\sqrt{3}}$

(36) $\cos\left(\frac{\frac{\pi}{4}}{2}\right) = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$