

Sum and difference

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}$$

① $\tan 75^\circ = \tan(45 + 30)$

$$\frac{\tan 45 + \tan 30}{1 - \tan 45 \cdot \tan 30} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \boxed{\frac{\sqrt{3}+1}{\sqrt{3}-1}}$$

Proofs

① $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ ✓

$$\underbrace{\cos \frac{\pi}{2}}_0 \cos x + \underbrace{\sin \frac{\pi}{2}}_1 \sin x$$

$\sin x$ ✓ $\sin(x \pm y) =$

② $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$ ±

$$\underbrace{\sin \frac{3\pi}{2}}_{-1} \cdot \cos x - \underbrace{\cos \frac{3\pi}{2}}_0 \cdot \sin x$$

$(-1)\cos x - 0(\sin x)$

③ $\tan(x+y) \cdot \tan(x-y) = \frac{\tan^2 x - \tan^2 y}{\dots}$

$$(3) \quad \tan(x+y) \cdot \tan(x-y) = \frac{\tan x - \tan y}{1 - \tan^2 x \tan^2 y}$$

$$\left(\frac{\tan x + \tan y}{1 - \tan x \tan y} \right) \cdot \left(\frac{\tan x - \tan y}{1 + \tan x \tan y} \right)$$

$$\frac{\tan^2 x - \tan^2 y}{1 - \tan^2 x \tan^2 y}$$

$$(4) \quad \cos 3x = \cos^3 x - 3 \sin^2 x \cdot \cos x$$

$$\cos(2x+x) = \overset{*}{\cos 2x} \cdot \overset{*}{\cos x} - \overset{*}{\sin 2x} \cdot \overset{*}{\sin x}$$

$\cos(x+x) \qquad \sin(x+x)$

$$(\cos x \cdot \cos x - \sin x \cdot \sin x) \cos x - \frac{(\sin x \cdot \cos x + \cos x \cdot \sin x) \sin x}{2 \sin x \cdot \cos x}$$

$$\cos^3 x - \sin^2 x \cdot \cos x - 2 \sin^2 x \cdot \cos x$$

$$\cos^3 x - 3 \sin^2 x \cdot \cos x$$