p.418) #16-26

(16) $\tan x + \sec x \checkmark = \dfrac{\cos x}{1-\sin x} = \dfrac{\cos x (1+\sin x)}{(1-\sin^2 x)} = \dfrac{\cos x + \sin x \cos x}{\cos^2 x} = \dfrac{\cos x}{\cos^2 x} + \dfrac{\sin x \cos x}{\cos^2 x}$

$$= \dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}$$

(17) $\cos^2 x - 1$
$$= \tan x + \sec x \checkmark$$

$$\overline{\cos x} = -\tan x \sin x \checkmark$$

$$\dfrac{-\sin^2 x}{\cos x} = \dfrac{-\sin x}{\cos x} \cdot \sin x = -\tan x \sin x \checkmark$$

(18) $\dfrac{\sec^2 \theta - 1}{\sin \theta} = \dfrac{\sin \theta}{1-\sin^2 \theta} \checkmark$

$$\dfrac{\tan^2 \theta}{\sin \theta} = \tan^2 \theta \cdot \dfrac{1}{\sin \theta} = \dfrac{\sin^2 \theta}{\cos^2 \theta} \cdot \dfrac{1}{\sin \theta} = \dfrac{\sin \theta}{\cos^2 \theta} = \dfrac{\sin^2 \theta}{1-\sin^2 \theta} \checkmark$$

(19) $(1-\sin\beta)(1+\csc\beta) = 1 - \sin\beta + \csc\beta - \sin\beta\csc\beta \checkmark$

$$1 - \sin\beta + \csc\beta - \sin\beta \csc\beta \checkmark$$

(20) $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = 2\csc^2 x \checkmark$

$$\dfrac{1+\cos x}{1-\cos^2 x} + \dfrac{1-\cos x}{1-\cos^2 x} = \dfrac{1+\cos x + 1 - \cos x}{\sin^2 x} = \dfrac{2}{\sin^2 x} = 2\csc^2 x \checkmark$$

(21) $(\cos t - \sin t)^2 + (\cos t + \sin t)^2 = 2 \checkmark$
$$\cos^2 t - 2\sin t \cos t + \sin^2 t + \cos^2 t + 2\sin t \cos t + \sin^2 t$$
$$\cos^2 t + \sin^2 t + \cos^2 t + \sin^2 t$$
$$1 \qquad + \qquad 1 \quad = \boxed{2} \checkmark$$

(22) $\sin^2\alpha - \cos^2\alpha = 1 - 2\cos^2\alpha \checkmark$

$$1 - \cos^2\alpha - \cos^2\alpha = 1 - 2\cos^2\alpha \checkmark$$

(23) $\dfrac{1+\tan^2 x}{\sin^2 x + \cos^2 x} = \sec^2 x \checkmark$

$$\dfrac{\sec^2 x}{1} = \sec^2 x \checkmark$$

(24) $\dfrac{1}{\tan\beta} + \tan\beta = \sec\beta\csc\beta \checkmark$

$$\dfrac{\cos\beta}{\cos\beta}\dfrac{\cos\beta}{\sin\beta} + \dfrac{\sin\beta}{\cos\beta}\dfrac{\sin\beta}{\sin\beta} = \dfrac{\cos^2\beta + \sin^2\beta}{\cos\beta\sin\beta} = \dfrac{1}{\cos\beta\sin\beta} = \dfrac{1}{\cos\beta} \cdot \dfrac{1}{\sin\beta} = \sec\beta\csc\beta \checkmark$$

(25) $\dfrac{\cos\beta}{1+\sin\beta} = \dfrac{1-\sin\beta}{\cos\beta} \checkmark$

$$\dfrac{\cos\beta(1-\sin\beta)}{(1+\sin\beta)(1-\sin\beta)} = \dfrac{\cos\beta(1-\sin\beta)}{1-\sin^2\beta} = \dfrac{\cos\beta(1-\sin\beta)}{\cos^2\beta} = \dfrac{1-\sin\beta}{\cos\beta} \checkmark$$

$$\frac{\cos\beta(1-\sin\beta)}{(1+\sin\beta)(1-\sin\beta)} = \frac{\cos\beta(1-\sin\beta)}{1-\sin^2\beta} = \frac{\cos\beta(1-\sin\beta)}{\cos^2\beta} = \frac{1-\sin\beta}{\cos\beta} \checkmark$$

㉖ $\dfrac{\sec x + 1}{\tan x} = \dfrac{\sin x}{1-\cos x} \checkmark$

$$\frac{\sec x}{\tan x} + \frac{1}{\tan x} = \sec x \frac{1}{\tan x} + \cot x = \sec x \cot x + \cot x$$

$$= \frac{1}{\cos x} \frac{\cos x}{\sin x} + \frac{\cos x}{\sin x}$$

$$= \frac{(1+\cos x)(1-\cos x)}{\sin x \quad (1-\cos x)}$$

$$= \frac{1-\cos^2 x}{\sin x(1-\cos x)} = \frac{\sin^2 x}{\sin x(1-\cos x)}$$

$$= \frac{\sin x}{1-\cos x} \checkmark$$