5.1 intro to trig identities

Reciprocal Identities

- $\csc x = \frac{1}{\sin x}$
- $\sec x = \frac{1}{\cos x}$
- $\sin x = \frac{1}{\csc x}$
- $\cos x = \frac{1}{\sec x}$
- $\tan x = \frac{1}{\cot x}$
- $\cot x = \frac{1}{\tan x}$

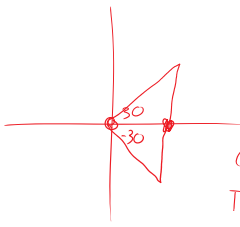
Quotient Identities

- $\tan x = \frac{\sin x}{\cos x}$
- $\cot x = \frac{\cos x}{\sin x}$

* Any manipulation of these

Odd - Even Identities

- $\sin(-x) = -\sin(x)$
  $\csc(-x) = -\csc x$
- $\cos(-x) = \cos x$
  $\sec(-x) = \sec x$
- $\tan(-x) = -\tan x$
  $\cot(-x) = -\cot x$

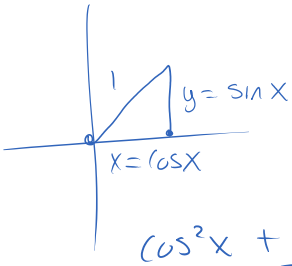


$\sin(30) = 1/2$
$\sin(-30) = -1/2$
$\cos(30) = \sqrt{3}/2$
$\cos(-30) = \sqrt{3}/2$
$\tan(30) = \frac{1}{\sqrt{3}}$
$\tan(-30) = -\frac{1}{\sqrt{3}}$

Pythagorean Identities

- $\cos^2 x + \sin^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $\cot^2 x + 1 = \csc^2 x$

* Any manipulation works.



$\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\sin(-x)$

$\frac{1}{\sin(-x)}$

(1) $\cot x \cdot \tan x$

$\frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x} = 1$

$\frac{1}{\tan x} \cdot \tan x$

(2) $\csc(-x) \cdot \sin(-x)$

$-\csc(x) \cdot -\sin x$

$\frac{1}{\sin x} \cdot \sin x = 1$

$\cos^2 x + \sin^2 x = 1$
$-\cos^2 x$

(3) $\frac{1 - \cos^2 x}{\sin x}$

$\sin^2 x = 1 - \cos^2 x$

$\frac{\sin^2 x}{\sin x} = \sin x$

$\frac{x^2}{x} = x$

$$\frac{\sin^2 x}{\sin x} = \sin x \qquad \frac{x^2}{x} = x$$

(4) $$\frac{\sec^2 x - \tan^2 x}{\cos^2 x + \sin^2 x} = \sec^2 x - \tan^2 x$$

$$= 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 = \sec^2 x - \tan^2 x$$

(5) $$\frac{(1 + \tan x)}{(1 + \cot x)} = \frac{\frac{\cos x}{\cos x}1 + \frac{\sin x}{\cos x}}{\frac{\sin x}{\sin x}1 + \frac{\cos x}{\sin x}} = \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}} \cdot \frac{\sin x}{\sin x + \cos x}$$

$$= \tan x$$

$$1 + \frac{2}{x}$$