Day 3 & 4 Notes

Monday, November 2, 2015 7:16 PM

Precalculus
3.2 Exponential and Logistic Growth Modeling

Name:
Period:

Exponential Population Model

If a population P is changing at a constant percentage rate r each year, then

$$P(t) = P_0(1+r)$$

where P_0 is the initial amount and r is expressed as a decimal,

and t is time in years. (months...etc)

$3\% \Rightarrow .03$

Finding an Exponential Function

1. The population of a small town near Rancho Cucamonga, CA has been growing by an average of 10.5% a year. The town was developed in 1950 with a population of 1255.

   $t=0$

   a. Write an exponential function modeling the growth or decay in terms of time, t.

   $$p(t) = 1255(1+.105)^t$$

   y x

   b. What will the population be in 2020?

   $$p(t) = 1255(1.105)^{70} \qquad 1,361,455$$

2. In 1990, college tuition averaged $4,000. Tuition has grown 8% each year.

   $t=0$

   a. Write an exponential function modeling the growth or decay in terms of time, t.

   $$T(t) = 4000(1.08)^t$$

   b. When will the average tuition reach $25,000?

   2013 - 2014

   $$\underbrace{25000}_{Y_1} = \underbrace{4000(1.08)^t}_{Y_2}$$

   $$6.25 = (1.08)^t$$

   $\approx 23.$

3. Congratulations! You just bought a new car! Unfortunately, information shows that your car will decrease in value by about 9% for each year you own the car. You paid $24,599 for your new car.

a. Write an exponential function modeling the growth or decay in terms of time, *t*.

$24599(1-.09)^t$

$C(t) = 24599(.91)^t$

b. When will your car be worth less than $10,000?

9.5 years

Practicing with Population Growth/Decay and ~~Half-Life~~ or Doubling/Tripling etc.

1. A population of guinea pigs that began with four doubles every four years.

a. Write an exponential function modeling the growth or decay in terms of time, *t*.

$p(t) = 4(2)^{t/4}$

b. How many guinea pigs will there be in 37 years?

$4(2)^{37/4}$

$2^{\wedge}(37/4)$

2435

2. A certain radioactive substance has a half-life of 30 days. Assume you begin with 100 grams of the substance.

a. Write an exponential function modeling the growth or decay in terms of time, *t*.

$p(t) = 100\left(\frac{1}{2}\right)^{t/30}$

b. How much of the substance is there after 200 days?

$100(.5)^{(200/30)}$

.98

c. When will there be less than 15g of the substance. Hint: set up the inequality and solve graphically.

$15 = 100\left(\frac{1}{2}\right)^{t/30}$

After 82.1 days

What about Logistic models??

Hinsdale Central's senior class has 1200 students. Seniors Peter, Patrick and Polly start a rumor that One Direction will be playing at this year's senior prom. The rumor spreads logistically so that

$$S(t) = \frac{c}{1 + a \cdot e^{-0.9t}}$$

models the number of seniors who have heard the rumor after t days.

a. What is the maximum number of seniors that can hear this rumor?

1200

$a \cdot b^x$

b. Determine the value of c.

c. How many seniors started the rumor?

3 → (0, 3)

d. Determine the value of a.

$(1+a)$ $\quad 3 = \frac{1200}{\cancel{1 + a \cdot e^{-.9(0)}}} \cancel{(1+a)}$ $\quad 1$

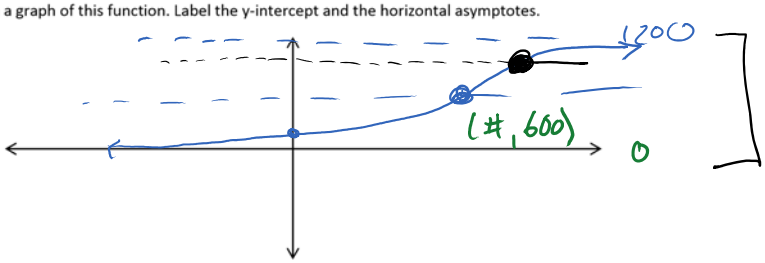
$1 + a = 400$

$a = 399$

e. Write the function for $S(t)$.

$S(t) = \frac{1200}{1 + 399 \cdot e^{-.9t}}$

f. Sketch a graph of this function. Label the y-intercept and the horizontal asymptotes.



g. How long does it take for 1000 students to hear the rumor?

$\underbrace{1000}_{Y_1} = \underbrace{\frac{1200}{1 + 399 \cdot e^{-.9t}}}_{Y_2}$

Finding a Logistic Function that models the given data.

Initial Value: 12	Limit to Growth: 60	Passing through (3, 49.5)

(0, 12)

$$f(x) = \frac{c}{1 + a \cdot b^x}$$

Step 1: Plug the "Limit to Growth" in for c.

Step 2: Since the "Initial Value" is the y-intercept, plug in (0, 12) for x and y and solve for a.

$$(1+a)\ 12 = \frac{60}{\cancel{1+a \cdot b^0}}\cancel{(1+a)}$$

$1 + a = 5$

$a = 4$

Step 3: We now have a and c, so plug in (3, 49.5) for x and y to find b.

$$(1+4b^3)\,49.5 = \frac{60}{\cancel{1+4 \cdot b^3}}\cancel{(1+4b^3)}$$

$$\frac{(1+4b^3)\,49.5}{49.5} = \frac{60}{49.5}$$

$1 + 4b^3 = 1.$

$-1 \qquad -1$

$b^3 = .053\ldots$

$b \approx .38$

Step 4: Write the function.

$$f(x) = \frac{60}{1 + 4 \cdot .38^x}$$