2.5 Notes

Monday, September 21, 2015 3:30 PM

$f(x) = x^5 + 2x +$ $\qquad$ $f(x) = 3x^2 + 2x + 8$

Precalculus
2.5 – Complex Zeros

Name:
Period:

Fundamental Theorem of Algebra

- A polynomial function of degree n has 'n' complex zeros.
- This includes imaginary and real zeros.
- Repeated zeros count each time they occur.

Linear Factorization Theorem

- If f(x) is a polynomial function of degree n, then f(x) has 'n' linear factors.
- Repeated factors count each time they occur.

Complex Conjugate Zeros

- If f(x) is a polynomial function with real coefficients and f(x) has a complex zero of $a+bi$ where $b \neq 0$, then $a-bi$ is also a zero of the polynomial.

$3+2i$
$3-2i$

(#1-2) Identify key features of the graph (x-intercepts) and then convert to standard form.

1. $f(x) = (x-5)(x-i\sqrt{2})(x+i\sqrt{2})$

Zeros of $f(x)$: $5,\ i\sqrt{2},\ -i\sqrt{2}$

x - intercepts of $f(x)$: $(5,0)$

Standard Form:

$(x-5)(x^2 - i^2\sqrt{4})$

$(x-5)(x^2+2)$

$\boxed{f(x) = x^3 - 5x^2 + 2x - 10}$

2. $f(x) = (x+i)(x-i)(x-(1+i\sqrt{2}))(x-(1-i\sqrt{2}))$

$(x-1-i\sqrt{2})(x-1+i\sqrt{2})$

Zeros of $f(x)$: $i,\ -i,\ 1+i\sqrt{2},\ 1-i\sqrt{2}$

x - intercepts of $f(x)$: None

Standard Form:

$(x^2 - i^2)((x-1)(x-1) - i^2\sqrt{4})$

$(x^2+1)(x^2-2x+1+2)$

$(x^2+1)(x^2-2x+3)$

$f(x) = x^4 - 2x^3 + 4x^2 - 2x + 3$

(#3-4) Write a polynomial in factored form with a minimum degree from the following roots.

3. -3, 4, $2-i$

Zeros of $f(x)$: $-3,\ 4,\ 2-i,\ 2+i$

x - intercepts of $f(x)$: $-3,\ 4,$

Factored Form:

$f(x) = (x+3)(x-4)(x-(2-i))(x-(2+i))$

4. 1, $1+2i$, $1-i$

Zeros of $f(x)$: $1,\ 1+2i,\ 1-2i,\ 1-i,\ 1+i$

x - intercepts of $f(x)$: 1

Factored Form:

$f(x) = (x-1)(x-(1+2i))(x-(1-2i))\ldots$

Recall **The Fundamental Theorem of Algebra** ... a polynomial with a degree of ***n*** has exactly n zeros.

To fully factor a polynomial:

1. Use your graphing calculator to find any real, RATIONAL zeros/x-intercepts. You should not round any decimals – rational numbers only (integers or fractions only)!
2. Using the zeros (one at a time) perform synthetic division to break down the polynomial to a quadratic.
3. Solve the quadratic – either by factoring or using the Quadratic Formula.
4. Use all zeros – both real and imaginary – to create linear factors.

Example #1

Find all zeros of $f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$ and write $f(x)$ in its linear factorization (factored form).

$x = 1, -2, 4$

$$\begin{array}{r|rrrrrr} 1 & 1 & -3 & -5 & 5 & -6 & 8 \\ & & 1 & -2 & -7 & -2 & -8 \\ \hline -2 & 1 & -2 & -7 & -2 & -8 & 0 \\ & & -2 & 8 & -2 & 8 & \\ \hline 4 & 1 & -4 & 1 & -4 & 0 & \\ & & 4 & 0 & 4 & & \\ \hline & 1 & 0 & 1 & 0 & & \end{array}$$

$x^2 + 1 = 0 \qquad \sqrt{x^2} = \sqrt{-1} \qquad x = \pm i$

$$\boxed{f(x) = (x-1)(x+2)(x-4)(x+i)(x-i)}$$

Example #2

Find all zeros of $g(x) = x^3 - 10x^2 + 44x - 69$ and write $g(x)$ in its linear factorization (factored form).

$X = 3$

$$\begin{array}{r|rrrr} 3 & 1 & -10 & +44 & -69 \\ & & 3 & -21 & 69 \\ \hline & 1 & -7 & 23 & 0 \end{array}$$

$x^2 - 7x + 23 = 0$

$$\frac{7 \pm \sqrt{49 - 4(1)(23)}}{2(1)} = \frac{7 \pm \sqrt{-43}}{2} = \frac{7 \pm i\sqrt{43}}{2}$$

$$\boxed{g(x) = (x-3)\left(x - \left(\frac{7+i\sqrt{43}}{2}\right)\right)\left(x - \left(\frac{7-i\sqrt{43}}{2}\right)\right)}$$