Geo H 10.9
Notes

10.9 Arc Length

$1^\circ = \frac{1}{360}$ of a given Circle

$$\text{Arc Length} = \frac{\text{Arc Measure}}{360} \cdot C$$

* Arc Measure = Central ∡

$C = 2\pi r$ or πd

$A = \pi r^2$

Example 1: Given the circumference of a circle is 18π, find the length of:

a) 35° arc

b) 120° arc

Example 2: If the arc length of a circle with radius 10 is 5π, find the circumference and area of the circle and measure of the arc.

$A.L = \frac{m\widehat{AB}}{360} C$

$5\pi = \frac{m\widehat{AB}}{\cancel{360}_{18}} \cdot \cancel{20}^{1}\pi$

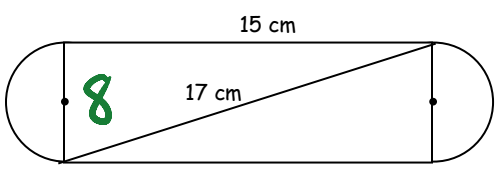
$5\pi = \frac{m\widehat{AB}\,\pi}{18}$

$90\pi = m\widehat{AB}\,\pi$

$\boxed{90 = m\widehat{AB}}$

Example 3: Find the perimeter of the following figures - EXACT answers.

a)

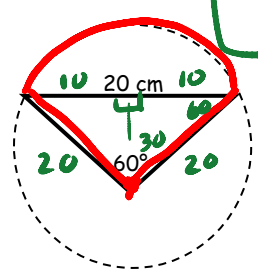


15

$C = 8\pi$

$\boxed{30 + 8\pi}$

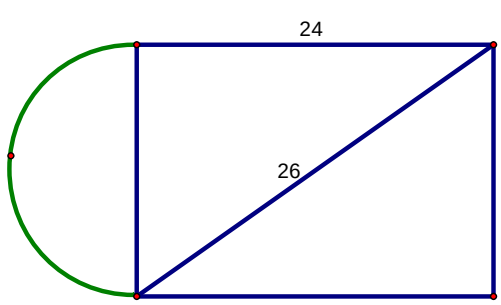
b)



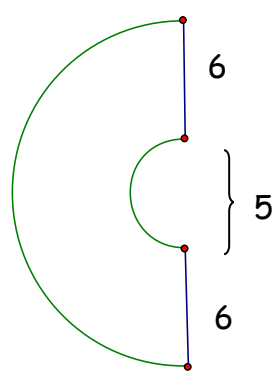
$\boxed{P = 40 + \frac{20\pi}{3}}$

$\frac{1}{6} \cdot 40\pi = \frac{20\pi}{3}$

c)

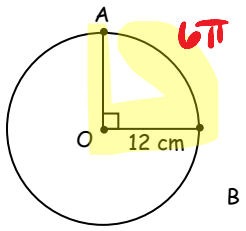


d)



Example 4: Find the perimeter of each "sector" - the region bounded by arc AB and the radii - EXACT answers.

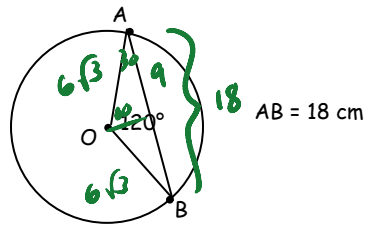
a)



$\frac{1}{4} \cdot 24\pi = 6\pi$

$\boxed{P = 6\pi + 24}$

b)



$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

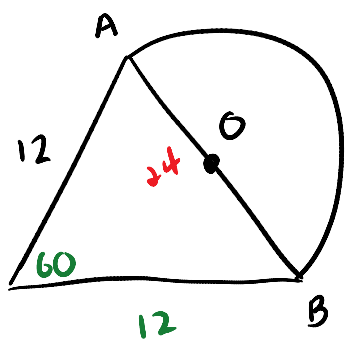
$x = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$

$\frac{1}{3} \cdot 12\pi\sqrt{3}$

Arc $= 4\pi\sqrt{3}$

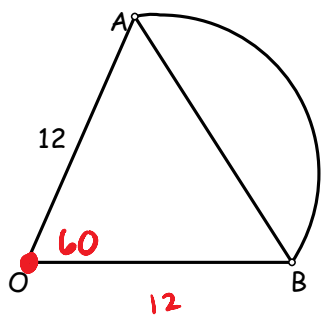
$\boxed{P = 12\sqrt{3} + 4\pi\sqrt{3}}$

Example 5: Find arc length of AB. The triangles are always equilateral and O is the center of the circle. (Note: none of the diagrams are to scale)



$C = 24\pi$

$\boxed{\overset{\frown}{AB} = 12\pi}$



$C = 24\pi$

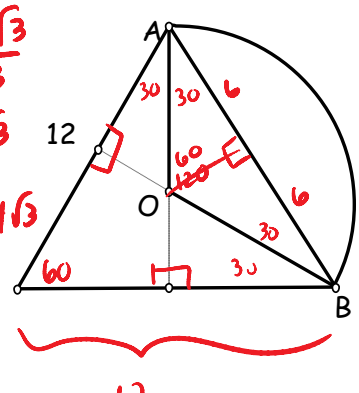
$\frac{1}{6} \cdot 24\pi = \boxed{4\pi}$

$x\sqrt{3} = 6$

$x = \frac{6\sqrt{3}}{3}$

$x = 2\sqrt{3}$

radius: $4\sqrt{3}$



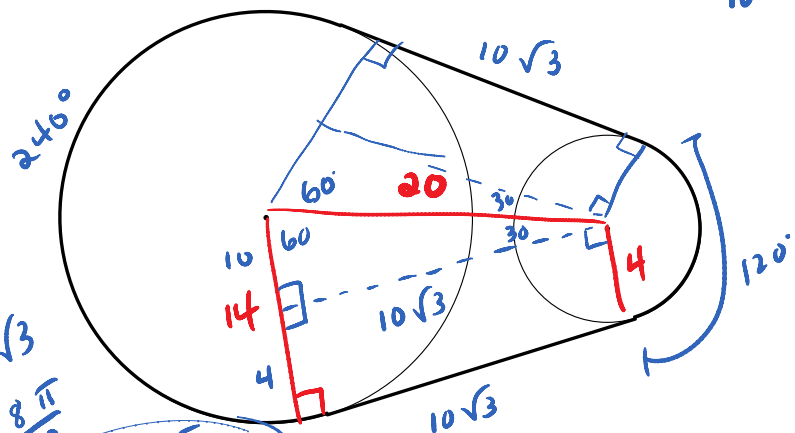
$C = 8\sqrt{3}\pi$

$\frac{1}{3}\, 8\sqrt{3}\pi = \boxed{\frac{8\sqrt{3}\pi}{3}}$

Example 6: Two pulleys are connected by a belt. The pulleys have radii of 14cm and 4cm. The distance between their centers is 20 cm. Find the total length of the belt needed to go around these two pulleys.

$10^2 + x^2 = 20^2$

$x = 10\sqrt{3}$



$C_{large} = 28\pi$

$A\,L = \frac{2}{3} \cdot 28\pi = \frac{56\pi}{3}$

$C_{small} = 8\pi$

$A.L = \frac{1}{3} \cdot 8\pi = \frac{8\pi}{3}$

$P = 10\sqrt{3} + 10\sqrt{3} + \frac{56\pi}{3} + \frac{8\pi}{3}$

$\sqrt{3} + \frac{64\pi}{3}$