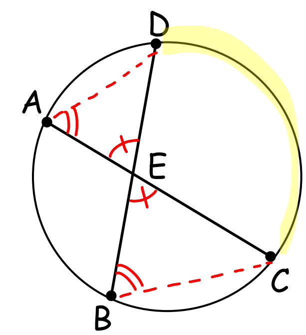
The Power Theorems

In problem #22 in section 10.6, we found similar triangles in the figure below.
They were: $\triangle DAE \sim \triangle CBE$



We could write the proportion of similar sides:

$$\frac{AE}{EB} = \frac{DE}{CE}$$

Using the Means-Extremes Product Theorem, we have:

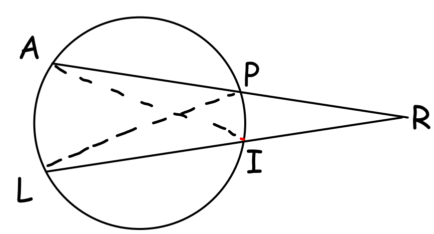
$$AE \cdot CE = EB \cdot DE$$

This gives us the CHORD- CHORD POWER THEOREM
If 2 chords of a circle intersect inside the circle, the product of the measures of the segments of one chord is equal to the product of the measures of the segments of the other chord.

In other words, "part · part = part · part"

What similar triangles can be found in this diagram?

$\triangle AIR \sim \triangle LPR$



We could write the proportion of similar sides:

$$\frac{AR}{LR} = \frac{IR}{PR}$$

Using the Means-Extremes Product Theorem, we have:

$$AR \cdot PR = LR \cdot IR$$

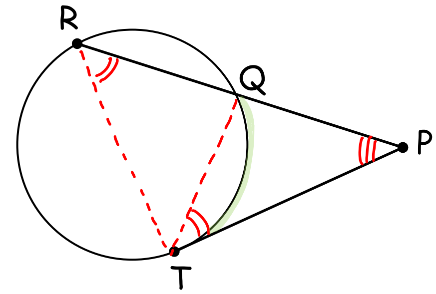
This gives us the SECANT- SECANT POWER THEOREM
If 2 secant segments are drawn from an external point to a circle, then the product of the measures of one secant segment and its external part equals the product of the measures of the other secant segment and its external part.

In other words, "external · whole = external · whole"

Let's try this one more time!

What similar triangles can be found in this diagram?

$\triangle PTR \sim \triangle PQT$



We could write the proportion of similar sides:

$$\frac{PT}{PR} = \frac{QP}{PT}$$

Using the Means-Extremes Product Theorem, we have:

$$PT \cdot PT = PR \cdot QP$$

$$PT^2 = PR \cdot QP$$

This gives us the TANGENT- SECANT POWER THEOREM

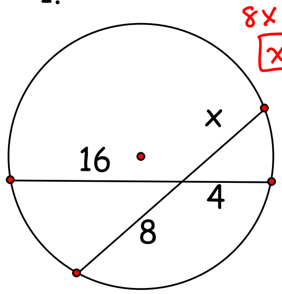
If a tangent segment and a secant segment are drawn from an external point to a circle, the square of the measure of the tangent segment is equal to the product of the measures of the entire secant segment and its external part.

In other words, "external · whole = external · whole"

Notice that this is the same as the secant-secant power theorem!

Examples: Find x

1.

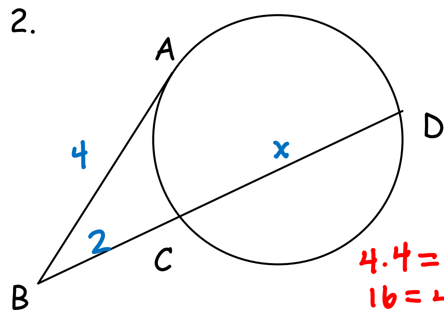


$8 \cdot x = 16 \cdot 4$

$8x = 64$

$\boxed{x = 8}$

2.



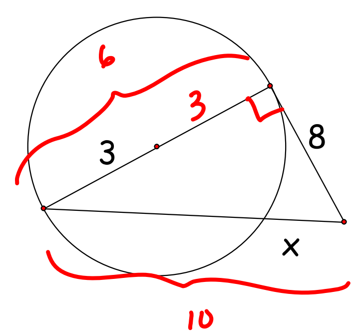
$4 \cdot 4 = 2(2 + x)$

$16 = 4 + 2x$

$12 = 2x$

$\boxed{6 = x}$

3.

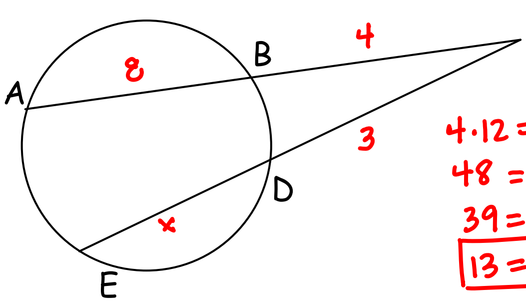


$8 \cdot 8 = x \cdot 10$

$64 = 10x$

$\boxed{6.4 = x}$

4. Given: AB = 8 Find: DE
BC = 4 CD = 3



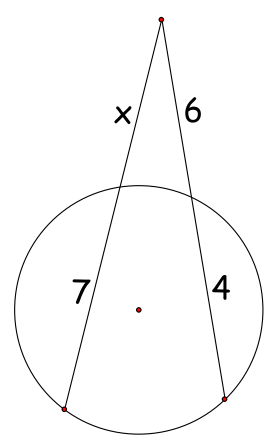
$4 \cdot 12 = 3(3 + x)$

$48 = 9 + 3x$

$39 = 3x$

$\boxed{13 = x}$

5.



$x(x+7) = 6 \cdot 10$

$x^2 + 7x = 60$

$x^2 + 7x - 60 = 0$

$12 \quad -5$

$(x+12)(x-5) = 0$

$x = \cancel{-12}, 5$